Absolute Space-time Theory with Variable Space-time and Gravitational Theory Established in Flat Space-time

Mei Xiaochun

(Department of Physics, Fuzhou University, E-mail: mxc001@163.com Tel:0086-591-87614214)

Abstract

The Einstein's theories of space-time and gravity are reconstructed thoroughly in this paper based on flat reference frame. The rational parts of the Einstein's theories are reserved while the irrational parts including space-time paradoxes and singularities are eliminated.

By transforming the geodesic equation described by the Schwarzschild solution of the Einstein's equation of gravitational field into flat reference frame for description, the revised formulas of the Newtonian gravitation can be obtained. Based on them, all experiments which support general relativity can also be described well, but there is no any space-time singularity in the theory. When the formula is used to discuss the problem of the universal expansion, the revised Hubble formula is obtained. It fits the conservative result of the high red-shift type Ia supernova well. Because the universal expansion is only controlled by gravitation in this theory, there exists no the accelerating expansion of the universe and the hypothesis of dark energy also becomes unnecessary. The problem of cosmic constant which has fazed physical circle for a long time can be ridded off thoroughly. A new formula of gravitational red shift can be obtained and the big red shifts of quasars can be explained well. The theory may also be used to explain so-called the Pioneer Anomaly which general relativity can not do.

It is proved by means of the dynamical effects of special relativity that velocity caused by accelerating process should be an absolute concept, instead of a relative concept. The influence of accelerating process should be considered in space-time theory. Besides the Newtonian absolute space-time theory with invariable space-time scales and the Einstein's relative space-time theory with variable space-time scales, there would exists the third space-time theory, i.e., the absolute space-time theory with variable space-time scales. In this way, theory of space-time can be consistent with the demand of the modern cosmology.

At present we only prove that inertial static mass and gravitational static mass are equivalent in the Eötvös type of experiments. By transforming the Schwarzschild solution into flat space-time to describe, we can deduce a conclusion consequentially that inertial moving mass and gravitational moving mass are equivalent. It can also be proved by means of the dynamic effect of special relativity that inertial forces and gravity are not equivalent locally and the principle of general relativity is actually untenable. By the coordinate transformations of the Kerr and Kerr-Newman solutions, the solutions for the static distributions of mass loop and double spheres with axial symmetry are obtained. The results indicate that space-time singularities in the Einstein's theory are caused actually by the descriptive method based on curved space-time. All of these show that the Einstein's theories of space-time and gravity should be revised.

Similar to electromagnetic theory, by introducing magnetic-like gravitation, a more rational theory of gravitation is established in flat space-time without any singularity. The theory may be consistent with quantum mechanics and may be renormalizable after being quantized. So it may provide a really reliable foundation for the unity of four interaction forces.

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Section 1 Absolute Space-time Theory with Variable Scales

----The third logically consistent and really rational space-time theory

Please see physics/0603017

Section 2 Rationality Problems of the Principles of Equivalence and General Relativity

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Section 3 Irrationality of the Axially Symmetrical Solutions of the Einstein's Equations of Gravitational Fields

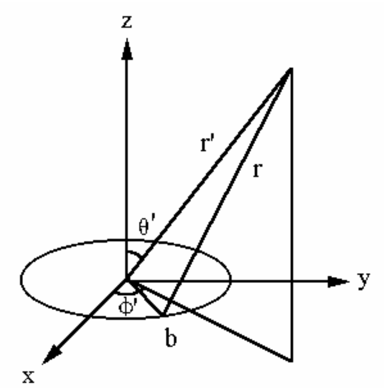
1. The static gravitational field of axial symmetry distribution of mass loop

Though the Einstein's theory of gravity obtained great success, we have only four experiments to verify for the simplest solution of gravitational fields with mass static spherical symmetry distribution up to now actually. Speaking strictly, we only prove it in the weak gravitational field of the sum. As for other solutions of the Einstein's equations of gravitational fields, we can neither obtain experimental supports nor find corresponding physical systems for them. Facing so many forms of material distributions and comparing with so many experimental verifications of the Newtonian theory of gravity and quantum mechanics as well as special relativity, it is far not enough for the Einstein's theory of gravity.

By means of the coordinate transformations of the Kerr solution with double parameters and the Kerr-Newman solution with three parameters of the Einstein's equation of gravitation field with axial symmetry, the gravitational field equation's solutions for the static mass distributions of thin loop and double spheres can be obtained. The results show that no matter what are the masses and densities of loop and double spheres, space's curvatures in the centre points of loop and the double sphere's connecting line are infinite. The singularity points are completely exposed in vacuum and space curvatures nearby the singularity points and the surface of loop and double spheres are also highly curved even though the gravitational fields are very weak. It is obvious that all of them are actually impossible. The results show that the space-time singularities appearing in the Einstein's theory of gravity are not owing to high density and huge mass's distributions. They are actually caused by the Einstein's theory itself and have nothing to do with real world. The so-called black holes, white holes and wormholes with space-time singularity are something illusive, not existing in nature actually. All of these results indicate that the Einstein's theory of gravity can not bet a universally suitable one. Physicists would be clear-headed for the Einstein's theory of gravity. It is unadvisable for physicists to lose their judgment ability only by the great authority of Einstein and the beautiful form of the theory.

The gravitational problem of mass thin loop distribution is discussed at first. As shown in Fig. 3.1, a thin loop with mass M and radium b is placed on the x-y plane. The centre of loop is located in the original point of coordinate system. The loop is thin enough so that its cross section can be regarded zero comparing with its perimeter. It will be seen later that if the cross section of loop is not zero, the result is also the same in essence. Because the static mass distribution of thin loop is with axial symmetry, in light of the Einstein's theory of gravity, the metric tensor of gravitational field has nothing to do with time t and coordinate φ, so the four dimension line element can be written as

$$ds^2 = g_{00}(r,\theta)dt^2 + g_{11}(r,\theta)dr^2 + g_{22}(r,\theta)r^2 d\theta^2 + +g_{33}(r,\theta)r^2 \sin^2 \theta d\varphi^2 \tag{3.1}$$



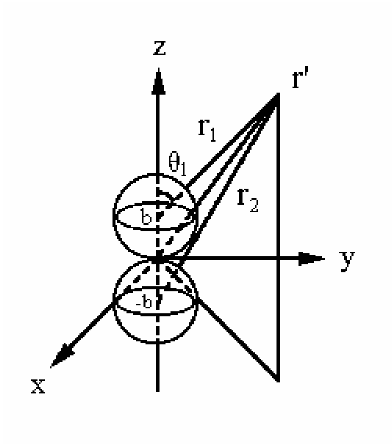


Fig. 3.1 Axial symmetry distribution of mass loop Fig. 3.2 Axial symmetry distribution of double spheres

If space-time is flat, we have $g_{\mu\nu} = 1$. By taking the coordinate transformations $t = t'$, $\varphi = \varphi'$, $r \to r' = r'(r,\theta)$, $\theta \to \theta' = \theta'(r,\theta)$, we can rewrite Eq.(3.1) as

$$ds^2 = g'_{00}(r',\theta')dt'^2 + g'_{11}(r',\theta')dr'^2 + g'_{22}(r',\theta')\ r'^2 d\theta'^2$$

$$+ g'_{33}(r',\theta')\ r'^2 \sin^2 \theta' d\varphi'^2 + g_{12}(r',\theta')r'dr'd\theta' \tag{3.2}$$

Two formulas above can be used to describe the gravitational field of thin loop. By putting the metrics above into the Einstein's equation of gravitational field, we can obtain the concrete forms of $g_{\mu\nu}$ in principle. But it is difficult to solve the equation of gravitational fields directly based on Eq.(3.1) or (3.2). On the other hand, there are two independent parameters M and b for the axial symmetry distribution of thin loop. There is also a ready-made solution of gravitational field's equation with axial symmetry and double parameters, i.e., the Kerr solution. If the solutions of the Einstein's equations of gravitational fields are unique, we can obtain the solution of static mass distribution of thin loop by means of the coordinate transformations of the Kerr solution. We have no other selection besides it. This method is discussed below.

The Kerr solution is [10]

$$ds^2 = \left(1 - \frac{2\alpha\, r}{r^2 + \beta^2 \cos^2\theta}\right)dt^2 - \frac{r^2 + \beta^2\cos^2\theta}{r^2 + \beta^2 - 2\alpha\, r}dr^2 - \left(1 + \frac{\beta^2}{r^2}\cos^2\theta\right)r^2 d\theta^2$$

$$-\left[1 + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2 \sin^2\theta}{r(r^2 + \beta^2\cos^2\theta)}\right]r^2 \sin^2 d\varphi^2 + 2\frac{2\alpha\beta\sin\theta}{r^2 + \beta^2\cos^2\theta} r\sin\theta\, dtd\varphi \tag{3.3}$$

At present, the Kerr metric is regarded as the solution of a revolving sphere's gravitational field，in which parameters $\alpha = GM$, $\beta = J/M$ is unit angle momentum ($c = \hbar = 1$). As for the static mass distribution of thin loop, the meanings of parameters α and β are discussed below. Because the metric (3.3) contains a cross item $dtd\varphi$ relative to time, it is a solution of dynamic state. In the static mass distribution problem, this item does not exist. We can remove it by the diagonalization of metric tensors. Because only the items relative to dt and $d\varphi$ should be taken into account, we can let

$$\begin{bmatrix} g_{00} & g_{30} \\ g_{03} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{2\alpha r}{r^2 + \beta^2\cos^2\theta} & \dfrac{2\alpha\beta\sin\theta}{r^2 + \beta^2\cos^2\theta} \\ \dfrac{2\alpha\beta\sin\theta}{r^2 + \beta^2\cos^2\theta} & -1 - \dfrac{\beta^2}{r^2} - \dfrac{2\alpha\beta^2\sin^2\theta}{r(r^2 + \beta^2\cos^2\theta)} \end{bmatrix} \tag{3.4}$$

Form the eigen equation

$$\begin{bmatrix} g_{00} - \lambda & g_{03} \\ g_{30} & g_{33} - \lambda \end{bmatrix} = 0 \tag{3.5}$$

we can get

$$\lambda_1 = \frac{1}{2}(g_{00} + g_{33} + \sqrt{(g_{00} - g_{33})^2 + 4g_{03}g_{30}})$$

$$= \frac{1}{2}\left[\left(2 - \frac{2\alpha\, r}{r^2 + \beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2 + \beta^2\cos^2\theta)}\right)^2 + \right.$$

$$\left. + \frac{16\alpha^2\beta^2\sin^2\theta}{(r^2 + \beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}} - \frac{1}{2}\left[\frac{2\alpha\, r}{r^2 + \beta\cos^2\theta} + \frac{\alpha^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2 + \beta^2\cos^2\theta)}\right] \tag{3.6}$$

$$\lambda_2 = \frac{1}{2}(g_{00} + g_{33} - \sqrt{(g_{00} - g_{33})^2 + 4g_{03}g_{30}})$$

$$\left. + \frac{16\alpha^2\beta^2\sin^2\theta}{(r^2 + \beta^2\cos^2\theta)^2}\right]^{\frac{1}{2}} + \frac{1}{2}\left[\frac{2\alpha\, r}{r^2 + \beta\cos^2\theta} + \frac{\alpha^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2 + \beta^2\cos^2\theta)}\right] \tag{3.7}$$

Therefore, we take the transformation (In this case r and θ are regarded as constants.)

$$dt' = \frac{-g_{00} + \lambda_2}{\lambda_2 - \lambda_1}dt - \frac{g_{30}}{\lambda_2 - \lambda_1} r\sin\theta d\varphi \tag{3.8}$$

$$d\varphi' = \frac{1}{r\sin\theta}\frac{g_{00} - \lambda_2}{\lambda_2 - \lambda_1}dt - \frac{g_{30}}{\lambda_2 - \lambda_1}d\varphi \tag{3.9}$$

Eq.(3.3) can be transformed into the diagonal form with

$$ds^2 = \frac{1}{2}\left\{\left[\left(2 - \frac{2\alpha\ r}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2\right.\right.$$

$$\left.\left. + \frac{16\alpha^2\beta^2\sin^2\theta}{\left(r^2+\beta\cos^2\theta\right)^2}\right]^{\frac{1}{2}} - \frac{2\alpha\ r}{r^2+\beta^2\cos^2\theta} - \frac{\beta^2}{r^2} - \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right\}dt'^2$$

$$-\frac{1}{2}\left\{\left[\left(2 - \frac{2\alpha r}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2 + \frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)}\right]^2\right.$$

$$-\frac{r^2+\beta^2\cos^2\theta}{r^2+\beta^2-2\alpha\ r}dr^2 - \left(1+\frac{\beta^2}{r^2}\cos^2\theta\right)r^2 d\theta^2$$

$$-\frac{1}{2}\left\{\left[\left(2 - \frac{2\alpha r}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2 + \frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)}\right]^2\right.$$

$$\left. + \frac{2\alpha\ r}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right\}r^2\sin^2\theta\ d\varphi'^2 \tag{3.10}$$

The formula above has the form of Eq.(3.1), so it can be used to describe the gravitational field of static mass distribution of thin loop.

On the other hand, as we known that only by comparing with the Newtonian theory in weak gravitational fields, the solution of the Einstein's equation of gravitational field can determined, otherwise the integral constants can not be decided so that the solution is meaningless. According to this principle in the general theory of relativity, we have relation

$$g_{00} = 1 + 2\psi \tag{3.11}$$

Here ψ is the Newtonian potential of thin loop. Now let's discuss its concrete form. As shown in Fig1, suppose the coordinates of observation point are $x_0 = r'\sin\theta'\cos\phi'$, $y_0 = r'\sin\theta'\sin\phi'$, $z_0 = r'\cos\theta'$. The coordinates of loop at a certain point are $x = b\cos\phi$, $y = b\sin\phi$, $z = 0$. The distance between these two points is

$$r = \sqrt{(x_0-x)^2+(y_0-y)^2+(z_0-z)^2} = \sqrt{r'^2+b^2-2r'b\sin\theta'\cos(\phi-\phi')} \tag{3.12}$$

For symmetry and simplification, we can take $\phi' = 0$, so the Newtonian potential of thin loop is

$$\psi = -\int\frac{Gdm}{r} = -\int_0^{\pi}\frac{2G\rho\ bd\phi}{\sqrt{r'^2+b^2-2r'b\sin\theta'\cos\phi}} \tag{3.13}$$

Here M , ρ and b are the mass, line density and radius of thin loop individually. Let $\phi = \pi - \phi'$, $d\phi' = -d\phi,\ \ -\cos\phi = \cos\phi' = 1 - 2\sin^2(\phi'/2)$, and put them into the formula above, we get

$$\psi = \int_{\pi}^{0} \frac{2G\rho\, bd\phi'}{\sqrt{r'^2 + b^2 + 2r'b\sin\theta - 4r'b\sin\theta'\sin^2(\phi'/2)}} \tag{3.14}$$

Then let $\phi'' = \phi'/2$ again, the formula above can written as

$$\psi = -\int_{0}^{\pi/2} \frac{4G\rho\, bd\phi''}{\sqrt{r'^2 + b^2 + 2r'b\sin\theta' - 4r'b\sin\theta'\sin^2\phi''}}$$

$$= -\frac{4G\rho\, b}{\sqrt{r'^2 + b^2 + 2r'b\sin\theta'}} \int_{0}^{\pi/2} \frac{d\phi''}{\sqrt{1 - k^2\sin^2\phi''}} \tag{3.15}$$

In the formula, $k^2 = 4r'b\sin\theta/(r'^2 + b^2 + 2br'\sin\theta)$. Let

$$K(k^2) = \int_{0}^{\pi/2} \frac{d\phi''}{\sqrt{1 - k^2\sin^2\phi''}}$$

(3.16)

It is just the first kind of ellipse function with $k^2 < 1$ when $r' \to \infty$. So we have

$$K(k^2) = \frac{\pi}{2}\left(1 + \frac{1}{4}k^2 + \frac{9}{64}k^4 \cdots\right) = \frac{\pi}{2}\left(1 + \frac{b\sin\theta'}{r'} + \frac{9b^2\sin^2\theta'}{4r'^2}\cdots\right) \tag{3.17}$$

On the other hand, when $r' >> b$, we have

$$\frac{1}{\sqrt{r'^2 + b^2 + 2r'b\sin\theta'}} = \frac{1}{r'}\left[1 - \frac{b\sin\theta'}{r'} + \frac{b^2\left(3\sin^2\theta' - 1\right)}{2r'^2} + \cdots\right] \tag{3.18}$$

Put them into Eq.(3.15), and let $2\pi\rho\, b = M$, we get

$$\psi = -\frac{GM}{r'}\left[1 - \frac{b^2\left(2 - 11\sin^2\theta'\right)}{4r'^2} + \cdots\right] \tag{3.19}$$

On the other hand, we can develop $g_{\mu\nu}$ into the power series of $1/r$ and write Eq.(3.10) as

$$ds^2 = \left(1 - \frac{2\alpha}{r} + \frac{2\alpha\beta^2\cos^2\theta}{r^3} + \cdots\right)dt'^2$$

$$-\left(1 + \frac{2\alpha}{r} - \frac{2\alpha + \beta^2\sin^2\theta}{r^2} + \frac{12\alpha\beta^2 - 48\alpha^3}{r^3} + \cdots\right)dr^2$$

$$-\left(1 + \frac{\beta^2}{r^2}\cos^2\theta\right)r^2 d\theta^2 - \left(1 + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r^3}\cdots\right)r^2\sin^2\theta\, d\varphi'^2$$

(3.20)

When $r \to \infty$, comparing the items with r^{-1} order in g_{00} and ψ, we get according to Eq.(3.11)

$$1 - \frac{2\alpha}{r} = 1 - \frac{2GM}{r'} \tag{3.21}$$

Let $\alpha = GM$, we get $r = r'$. However, the formula above is only the corresponding relation when the mass is concentrated at the center point of thin loop. It is not the gravitational potential of thin loop. In order to obtain the potential of thin loop, we should consider higher order items. There are no items with r^{-2} order in both Eq.(3.19) and (3.20). By considering the items containing r^{-3} order, we have

$$1-\frac{2GM}{r}+\frac{2GM\beta^2\cos^2\theta}{r^3}=1-\frac{2GM}{r'}+\frac{2GMb^2\left(0.5-2.75\sin^2\theta'\right)}{r'^3} \tag{3.22}$$

Thus we see that the function forms are different if high order items are considered. It means that the Einstein's theory of gravity can not asymptotically coincide with the Newtonian theory automatically in general. In order to let them asymptotically coinciding to each other, further transformation is needed. Because constant β has the dimension of length, in the problem of thin loop, we can let $\beta=b$. Because we always have $\cos^2\theta\geq 0$, but may have $0.5-2.75\sin^2\theta'<0$ in some cases, so in general we have $0.5-2.75\sin^2\theta'\neq\cos^2\theta$. But we can let $\theta=\theta'$. In this way, Eq.(3.22) becomes

$$\frac{1}{r}-\frac{b^2\cos^2\theta'}{r^3}=\frac{1}{r'}-\frac{b^2\left(0.5-2.75\sin^2\theta'\right)}{r'^3} \tag{3.23}$$

Let $A=b^2\cos^2\theta'$, $B=b^2(0.5-2.75\sin^2\theta')$. Because of $r'>>b$, the only real solution of the third order equation above is

$$\frac{1}{r}=\left[\frac{1}{2Ar'^3}\left(B-r'^2\right)+\frac{i}{2A^{3/2}r'^3}\sqrt{4r'^6-A\left(B-r'^2\right)^2}\right]^{\frac{1}{3}}$$

$$+\left[\frac{1}{2Ar'^3}\left(B-r'^2\right)-\frac{i}{2A^{3/2}r'^3}\sqrt{4r'^6-A\left(B-r'^2\right)^2}\right]^{\frac{1}{3}}$$

$$=\left(a+ib\right)^{1/3}+\left(a-ib\right)^{1/3}=2Q\cos\left(\delta/3\right) \tag{3.24}$$

Here $a=(B-r'^2)/(2Ar'^3)$, $b=\sqrt{4r'^6-A(B-r'^2)^2}/(2A^{3/2}r'^3)$, $Q=(a^2+b^2)^{1/6}$, $\delta=tg(b/a)$. In this way, we can write $r=r(r',\theta')$ and obtain

$$dr=\frac{dr}{dr'}dr'+\frac{dr}{d\theta'}d\theta'=T(r',\theta')dr'+V(r',\theta')d\theta' \tag{3.25}$$

The concrete forms of functions $T(r',\theta')$ and $V(r',\theta')$ are unimportant, so the concrete forms are not written out here. Now put the relation into Eq.(3.10), the solution of gravitational equation of thin loop is reached with the form of Eq.(3.2)

$$ds^2=\frac{1}{2}\left\{\left[\left(2-\frac{2\alpha\ r}{r^2+b^2\cos^2\theta'}+\frac{b^2}{r^2}+\frac{2\alpha b^2\sin^2\theta'}{r(r^2+b^2\cos^2\theta')}\right)^2\right.\right.$$

$$\left.\left.+\frac{16\alpha^2b^2\sin^2\theta'}{\left(r^2+b\cos^2\theta'\right)^2}\right]^{\frac{1}{2}}-\frac{2\alpha\ r}{r^2+b^2\cos^2\theta}-\frac{b^2}{r^2}-\frac{2\alpha b^2\sin^2\theta'}{r(r^2+b^2\cos^2\theta')}\right\}dt'^2$$

$$-\frac{r^2+b^2\cos^2\theta'}{r^2+b^2-2\alpha\ r}T^2(r',\theta')dr'^2-\left[\left(1+\frac{b^2}{r^2}\cos^2\theta'\right)\frac{r^2}{r'^2}\right.$$

$$+\frac{r^2+b\cos^2\theta'}{r^2+b^2-2\alpha b}\frac{V^2(r',\theta')}{r'^2}\Bigg]r'^2d\theta'^2-\frac{r^2}{2r'^2}\Bigg\{\Bigg[\Bigg(2-\frac{2\alpha r}{r^2+b^2\cos^2\theta'}$$

$$+\frac{b^2}{r^2}+\frac{2ab^2r\sin^2\theta'}{r\left(r^2+b^2\cos^2\theta'\right)^2}\Bigg)^2+\frac{16\alpha^2b^2\sin^2\theta'}{(r^2+b^2\cos^2\theta')^2}\Bigg]^{\frac{1}{2}}$$

$$+\frac{2\alpha\ r}{r^2+b^2\cos^2\theta'}+\frac{b^2}{r^2}+\frac{2\alpha b^2\sin^2\theta'}{r(r^2+b^2\cos^2\theta')}\Bigg\}r'^2\sin^2\theta'\ d\varphi'^2$$

$$-\left[2\frac{r^2+b^2\cos^2\theta'}{r^2+b^2-2\alpha\ r}\frac{T(r',\theta')V(r',\theta')}{r'}\right]r'dr'd\theta' \qquad (3.26)$$

Here $r=r(r',\theta')$. Because Eqs. (3.24) and (3.26) are too complex, we discuss the gravitational field of thin loop from relation (3.23) directly. It is known that when $r'=0$ we have $r=0$. So when $r'=0$, we have $g_{00}\to\infty$, $g_{22}\to\infty$, and $g_{33}\to\infty$. The results show that a singularity appears in the centre point of thin loop. This singularity is exposed in vacuum completely, no matter how much the mass of thin loop is, big or small. In the nearby region of the center point, space is also high curved. This result is absolutely absurd for it obviously violate common knowledge. It does not like the Schwarzschild solution in which the singularity is hided in the center of big mass so that it can not be observed directly. In this case, it seems that physicists can bear the existence of singularity. But we can not bear the singularity to appear in the centre point of a thin loop with a small mass without being covered.

Besides, space nearby thin loop's surface is also high curved. Take $\alpha\to 0$ and $\theta'=\pi/2$ on the surface of thin loop. In this case, Eq.(3.26) becomes

$$ds^2=dt'^2-\frac{r^2}{r^2+b^2}T^2(r',\theta')dr'^2-\left[\frac{r^2}{r'^2}+\frac{r^2V^2(r',\theta')}{r'^2(r^2+b^2)}\right]r'^2d\theta'^2$$

$$-\left(\frac{r^2}{r'^2}+\frac{b^2}{r'^2}\right)r'^2\sin^2\theta'\ d\varphi'^2-\frac{2r^2}{r'(r^2+b^2)}T(r',\theta')V(r',\theta')r'dr'd\theta' \qquad (3.27)$$

Let $b=0.67$, Eq.(3.23) becomes

$$\frac{1}{r}=\frac{1}{r'}+\frac{1}{r'^3} \qquad \text{or} \qquad r=\frac{r'^3}{r'^2+1} \qquad (3.28)$$

$$T(r',\theta')=\frac{r'^3+3r'^2}{(r'^2+1)^2} \qquad V(r',\theta')=0 \qquad (3.29)$$

Taking $r'=b=0.67$, we have $r=0.21,\ T=1$. Put them into Eq.(3.27), we obtain $g'_{11}=-0.09$, $g'_{22}=-0.01$, $g'_{33}=-1.01$. It is obvious that the space nearby the surface of thin loop is high curved. This does not agree with practical situation completely. On the surface of thin loop, for such a weak gravitational field, the space should be nearly flat with $g'_{11}=g'_{22}=g'_{33}\approx-1$. Because these are measurable quantities, it can be said that the Einstein's theory of gravity is unsuitable for the problem of mass distribution of thin loop.

On the other hand, let $r^2+b^2-2\alpha\ r=0$ in Eq.(3.26), we have $r=\alpha\pm\sqrt{\alpha^2-b^2}$. By taking

$M \sim 1Kg$ for weak field, we have $\alpha \sim GM/c^2 = 7.41\times10^{-28}m$. Because of $\alpha << b$, r is not a real number in this case. The second singularity decided by the relation $r^2 + b^2 - 2\alpha\, r = 0$ in Eq.(3.26) does not exist in general. But according to the Newtonian theory, according to Eq.(3.13) at the loop's center point $r' = 0$, gravitational potential is a limit constant with

$$\psi = -\int_0^{\pi} 2G\rho\, d\phi = -2\pi\, G\rho = -\frac{GM}{b} \tag{3.30}$$

So gravity at the center point is zero. This agrees with practical situation. Therefore, it should be noted that in order to decide the integral constants of the solutions of the Einstein's equation of gravitational fields, we have to let the Einstein's theory asymptotically coinciding with the Newtonian theory under the condition of weak fields with $r \to \infty$. Otherwise we can not compare the Einstein's theory with the Newtonian one, so that we can not determinate the validity of the Einstein's gravity theory. But in the problem of static axial symmetry distribution of thin loop mass, singularity appears at the center point of loop in vacuum so that the theory becomes meaningless. On the region nearby the center and surface of loop, space-time is high curved. All of these do not agree with practical situations.

The situation when the cross section of thin loop is not zero is discussed below. In this case, the gravitation field is with three parameters. The third is the radium of loop's cross section. On the other hand, as we known that the Kerr-Newman metric is one with axial symmetry and three parameters [11]. At present, it is used to describe the external gravitational field of charged revolving sphere. If the solution of the Einstein's equation of gravitational field with three parameters and axial symmetry is unique, through the coordinate transformation from the Kerr-Newman metric, we can also reach the gravitational field of loop with cross section. By the same method of diagonalization , we can write the Kerr-Newman metric as

$$\begin{aligned} ds^2 = \frac{1}{2}\Bigg\{\Bigg[\left(2 - \frac{2\alpha\, r - Q^2}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2 \\ + \frac{16\alpha^2\beta^2\sin^2\theta}{\left(r^2+\beta\cos^2\theta\right)^2}\Bigg]^{\frac{1}{2}} - \frac{2\alpha\, r - Q^2}{r^2+\beta^2\cos^2\theta} - \frac{\beta^2}{r^2} - \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\Bigg\} dt'^2 \\ - \frac{r^2+\beta^2\cos^2\theta}{r^2+\beta^2-2\alpha\, r - Q^2}dr^2 - \left(1+\frac{\beta^2}{r^2}\cos^2\theta\right)r^2 d\theta^2 \\ - \frac{1}{2}\Bigg\{\Bigg[\left(2 - \frac{2\alpha\, r - Q^2}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\right)^2 + \frac{16\alpha^2\beta^2\sin^2\theta}{(r^2+\beta^2\cos^2\theta)^2}\Bigg]^{\frac{1}{2}} \\ + \frac{2\alpha\, r - Q^2}{r^2+\beta^2\cos^2\theta} + \frac{\beta^2}{r^2} + \frac{2\alpha\beta^2\sin^2\theta}{r(r^2+\beta^2\cos^2\theta)}\Bigg\} r^2\sin^2\theta\, d\varphi'^2 \end{aligned} \tag{3.31}$$

Here constant Q is relative to the charge of sphere. When $r >> \alpha$ and $r >> \beta$, we can get from formula above

$$g_{00} = 1 - \frac{2\alpha}{r} + \frac{Q^2}{r^2} + \frac{2\alpha\beta^2\cos^2\theta^2}{r^3} + \cdots \tag{3.32}$$

On the other hand, when the area of thin loop's cross section is considered, the function form of the Newtonian potential is very complex. We can get the same conclusion by a simple estimate without accurate calculation. Suppose the radium of thin loop's cross section is h, when $r >> b$, $r >> h$ and $h \sim b$, we can always write the Newtonian gravitational potential of thin loop as

$$\psi = -\frac{GM}{r'}\left[1 + \frac{f_1(\theta', b, h)}{r'} + \frac{f_2(\theta', b, h)}{r'^2} + \cdots\right] \tag{3.33}$$

From discussion above, we can take $\theta = \theta'$, $\beta = b$. When $r \to \infty$, we can only remain the item containing r^{-2} and get

$$-\frac{GM}{r} + \frac{Q^2}{2r^2} = -\frac{GM}{r'} + \frac{f_1(\theta', b, h)}{r'^2} \tag{3.34}$$

Let $x = 1/r$, $x' = 1/r'$, we have from formula above

$$x = \frac{GM + \sqrt{(GM)^2 + 2Q^2\left(f_1 x'^2 - GMx'\right)}}{Q^2} \tag{3.35}$$

Put it into Eq.(3.31), we can get the metric of loop. It is easy to know from the formulas that when $r' = 0$ ($x' = \infty$), we have $r = 0$ ($x = \infty$). So it can also be seen from Eq.(3.33) that there exists still singularity at the center point of loop, and the singularity is also exposed in vacuum completely. Space nearby the center point and the surface of loop is also high curved. The situation is completely the same as that when the area of cross section of thin loop is not taken into account.

2. The static gravitational field of axial symmetry distribution of double spheres

The static gravitational field of mass double sphere is discussed below. As shown in Fig. 3.2, the masses and radius of both spheres are M and b individually. The centers of two spheres are at the points $\pm b$ of the z axis. The gravitational field of this axial symmetrical distribution with two parameters can also be obtained through the coordinate transformation of the Kerr solution. For this problem, the Newtonian potential of gravity is

$$\psi = -GM\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = -GM\left(\frac{1}{\sqrt{r'^2 + b^2 + 2br'\cos\theta}} + \frac{1}{\sqrt{r'^2 + b^2 - 2br'\cos\theta}}\right) \tag{3.36}$$

When $r' >> b$, we have

$$\psi = -\frac{2GM}{r'}\left(1 - \frac{b^2 - 3b^2\cos^2\theta}{2r'^2}\right) \tag{3.37}$$

From Eq.(3.11), we get relation

$$1 - \frac{2\alpha}{r} + \frac{2\alpha\beta^2\cos^2\theta}{r^3} = 1 - \frac{4GM}{r'} + \frac{2GMb^2(1 - 3\cos^2\theta')}{r'^3} \tag{3.38}$$

Let $\alpha = GM$, $\beta = b$ and $\theta = \theta'$, we have

$$\frac{1}{r}-\frac{b^2\cos^2\theta'}{r^3}=\frac{1}{r'}-\frac{b^2(1-3\cos^2\theta')}{2r'^3} \tag{3.39}$$

Let $A=b^2\cos^2\theta'$, $B=b^2(1-3\cos^2\theta')/2$, from Eq.(3.39), we can also obtain the formula similar to Eq.(3.24). Put it into Eq.(3.10), the metrics of mass double sphere distribution is also be obtained. For simplification, we can discuss directly from Eq.(3.39). When $r'=0$, we have $r=0$ so that $g_{00}\to\infty$, $g_{22}\to\infty$, $g_{33}\to\infty$ as well as $g_{23}\to\infty$ ($T\neq 0$, $V\neq 0$). The singularity appears at the point at which two spheres contact each other. Suppose $b=\sqrt{2}$ and $\theta=\pi/2$, Eq.(3.39) becomes

$$\frac{1}{r}=\frac{1}{r'}-\frac{1}{r'^3} \qquad \text{or} \qquad r=\frac{r'^3}{r'^2-1} \tag{3.40}$$

Let $M=1Kg$, i.e., the gravitational field is very weak so that we can let $\alpha\to 0$ in Eq.(3.26). The formula similar to Eq.(329) can be obtained with

$$T(r',\theta')=\frac{r'^4-3r'^2}{(r'^2-1)^2} \qquad V(r',\theta')=0 \tag{3.41}$$

Let $r'=2$ at some points on the surface of double spheres, we get $r=2.67$ and $T=0.44$. Put them into Eq.(3.27), we obtain $g_{11}=-0.34$, $g_{22}=-1.78$ and $g_{33}=-2.28$. It means that the space nearby the surfaces of two spheres is also high curved. It is also obvious that this result does not agree with practical situation completely. For this weak field, space should be nearly flat with $g_{11}=g_{22}=g_{33}\approx -1$. More serious problem is that when $r'<1$, according to Eq.(3.40), r becomes a negative number so that it is meaningless. So it can be said that the Einstein's theory of gravity is unsuitable for the problem of double spherical mass distribution.

In fact, there are many other axial symmetry distributions of static masses with double and three parameters. For example, three spheres are superposed one by one in a line, two cones are superposed with their cusps meeting together and hollow column and so do. All of their gravitational fields should be obtained by means of the coordinate transformations of the Kerr and Kerr-Newman solutions in principle. But as shown above, the same problems would be caused.

3. Discussion in general situations

Therefore, we can conclude from discussions above

1. If the axial symmetry solutions of the Einstein's equations of gravitational fields with double and three parameters are unique, the gravitational fields of the static mass axial symmetry distributions of thin loop and double spheres should be obtained by the coordinate transformations of the Kerr and Kerr-Newman solutions. However, these solutions can not coincide with practical situations. Therefore, the Einstein's theory of gravity can not be a universally suitable one. If the axial symmetry solutions of the Einstein's equations of gravitational fields with double and three parameters are not unique, i.e., there exist other solutions for the same Einstein's equation which can describe these problems well (even though they have not be founded now), the uniqueness which is a basic demand for a universal physical theory would be destroyed.

2. A great number of theories about space-time singularity, black holes, white holes and wormholes have established based on the general theory of relativity. In common viewpoint, these objects with space-time singularity are caused by the distributions of high density and huge masses. However, from

discussions above, even for the systems of thin loop and two small spheres, singularities would also appear based on the Einstein's theory of gravity. This fact shows that singularities are not caused actually by the high density and huge masses. They only exist in the Einstein's theory actually, not in nature. The singularity has nothing to do with real world. So-called black holes, white holes and wormholes with space-time singularity are actually illusive objects. This problem will be further discussed later.

3. As well-known that only by comparing with the Newtonian theory under the condition of weak fields, the solution forms of the Einstein's equations of gravitational fields can be finally determined. According to the present method, under the condition of weak field, let $h_{\mu\nu}$ to be a small quantity with

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3.42}$$

Here $\eta_{\mu\nu}$ is the Minkowski metric. In this case, the Einstein's equations become[12]

$$\Box^2 h_{\mu\nu} - \frac{\partial^2}{\partial x^\lambda \partial x^\mu} h^\lambda_\nu - \frac{\partial^2}{\partial x^\lambda \partial x^\nu} h^\lambda_\mu + \frac{\partial^2}{\partial x^\mu \partial x^\nu} h^\lambda_\lambda = -16\pi G S_{\mu\nu} \tag{3.43}$$

$$S_{\mu\nu} = T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^\lambda_\lambda \tag{3.44}$$

By introducing proper function ε^μ and taking coordinate transformations below

$$x^\mu \to x^\mu + \varepsilon^\mu\left(x^\mu\right) \qquad\qquad h_{\mu\nu} \to h_{\mu\nu} - \frac{\partial \varepsilon_\mu}{\partial x^\nu} - \frac{\partial \varepsilon_\nu}{\partial x^\mu} \tag{3.45}$$

the harmonious coordinate condition $g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$ can be satisfied, so that we have

$$\frac{\partial}{\partial x^\mu} h^\mu_\nu = \frac{1}{2}\frac{\partial}{\partial x^\nu} h^\mu_\mu \tag{3.46}$$

The energy and momentum tensors shown in Eq.(3.43) should also be transformed correspondingly. Put the relations into Eq.(3.43), the equation becomes

$$\Box^2 h_{\mu\nu} = -16\pi\, G S_{\mu\nu} \tag{3.47}$$

The equations has the solution similar to classical retarded potential with

$$h_{\mu\nu}(\vec{x},t) = \int \frac{4GS_{\mu\nu}(\vec{x}',t-r)}{r} d^3\vec{x}' \tag{3.48}$$

Here $r = |\vec{x} - \vec{x}'|$, $h_{00} = 2\psi$, ψ is the Newtonian potential. In this way, it seems that the Einstein's theory is proved to coincide with the Newtonian theory under the condition of weak field. However, by careful examination, we would find that the thing is not so simple. In order to reach Eq.(3.48), we have to introduced coordinate transformation (3.45). Meanwhile, the same transformation should be taken for Eq.(3.44). It means that the mass and energy's distribution form of original system would be changed and the original solution would be transformed into other one with different symmetry. In this way, the new solution (3.48) is meaningless for the original problem we wand to discuss. This problem will be discussed again in next paper, for it involves the rationality problem of the principle of general relativity. Here we only provide an example to show this conclusion. That is the so-called Kasner metric[13] for the solution of the Einstein's equation with infinite mass line (or column) distribution

$$ds^2 = r^{2a}dt^2 - dr^2 - r^{2b}d\varphi^2 - r^{2c}dz^2 \tag{3.49}$$

In general, we have $a \neq 0$, $b \neq 0$ and $c \neq 0$, otherwise the solution becomes the Minkowski metric of flat space-time. According to the Newtonian theory, for the same problem, the strength of gravitational field external line (or column) is

$$E = -\frac{G\rho}{r}$$

(3.50)

Here $\rho =$constant is line mass density. When r is big enough, E is small enough and the field can be considered as a weak one. In this case we have $g_{00} = r^{2a} = 1 + 2\psi$, or

$$\psi = \frac{1}{2}\left(r^{2a} - 1\right) \tag{3.51}$$

So when r is big enough, from formula above, we get

$$E = -\frac{d\psi}{dr} = -ar^{2a-1} \tag{3.52}$$

Comparing with the result of Newtonian theory according to Eq.(3.50), we should have

$$\frac{G\rho}{r} = ar^{2a-1} \tag{3.53}$$

However it is obvious that when $a \neq 0$, the function forms on the two sides of equation are completely different so that it is impossible to compare them. When $a = 0$, the right side of the formula is equal to zero but the left side is not, so the equation does not exist. In fact, when $r \to \infty$, we have

$$\lim_{r\to\infty} r^{2a} = \begin{cases} 0 & a > 0 \\ \infty & a < 0 \end{cases} \qquad \lim_{r\to\infty} r^{2b} = \begin{cases} 0 & b > 0 \\ \infty & b < 0 \end{cases} \qquad \lim_{r\to\infty} r^{2c} = \begin{cases} 0 & c > 0 \\ \infty & c < 0 \end{cases} \tag{3.54}$$

It is obvious that all metrics g_{00}, g_{22} and g_{33} in Eq.(4.49) can not be written as the forms of Eq.(3.42), so that we can not connect the Einstein's theory with the Newtonian one, i.e., the Einstein's theory can not coincide asymptotically with the Newtonian theory. Because we can not establish relations between constants G, ρ and a, b and c so that the constants a, b and c can not be determined, the solution (3.49) is meaningless actually for the problem of static mass line distribution. In fact, the Einstein's theory and the Newtonian theory are two completely different systems with completely different starting points. It is impossible for them to reach asymptotical consistent under the conditions of weak fields in general.

The common procedure to obtain the solutions of the Einstein's equation of gravity is firstly to simplify the metrics based on some space-time symmetry, then to solve the equation. Up to now, a lot of solutions have been founded. But many of them are considered meaningless in physics for no practical systems can be founded corresponding to them. However, the real situations may be that

1. Under the conditions of weak fields, the solutions of the Einstein's theory can not asymptotical coincide with the Newtonian theory. In this case, the integral constants in the solutions of the Einstein's equations can not be determined so that we can say that the Einstein's theory of gravity is actually unsuitable for these problems.

2. Though the solutions of the Einstein's equations of gravitational fields can coincide with the Newtonian theory in weak fields, but they are obviously irrational in general situations.

At present, if the results of the Newtonian theory and the Einstein's theory are different, we always consider that the Newtonian theory is wrong. But should we ask whether the Einstein's theory of gravity is alright? For weak gravitational fields, the Newtonian theory goes through so many verifications and can be considered basically correct. For the same problems under the condition of weak fields, if the Einstein's theory can not asymptotically coincide with the Newtonian theory, how can we always say that the Einstein's theory is wrong? Besides suspecting the correctness of the Einstein's theory, we have no other outlet. It is not a scientific attitude when we find that the Einstein's theory can not asymptotically coincide with the Newtonian theory for a certain problem, we only say that this solution of Einstein's equation is meaningless in physics then sent it away randomly. We can not help to ask that if these solutions are meaningless, where are the meaningful ones for the same problems?

Up to new, only the Schwarzschild solution obtained four verifications actually. Speaking strictly, the solution was proved effective only in the weak gravitational field of the sun. The verifications are too littler comparing with the Newtonian theory of gravity, quantum mechanics and special relativity. On the hand, under the condition of strong field, space-time singularity appears in the Schwarzschild solution. So it may be said that the correctness of the Schwarzschild solution is only a coincidence. It is unsuitable to regard the Einstein's theory of gravity as a foundational theory of interaction. As we know that though we need lots of proofs to verify a theory, only a proof can overthrow a theory sometimes. Physicists should keep their brains clear for the Einstein's theory of gravity. It is unadvisable for physicists to lose their judgment ability only by the great authority of Einstein and the beautiful form of the theory. Physics is an experimental science to pursue reality. The beauty of format is not main aim. Besides the Einstein's theory, there are many other theories of gravity now. But most of them are established based on the concept of curved space-time. Therefore, all of them are facing the same problems which exist in the Einstein's theory. Owing to the problems mentioned in the paper, we should survey the rationality of the Einstein's theory.

Because in the weak gravitational field of the sun, the Einstein's theory of gravity is the most simple and effective one, it would have some rationality. As shown in next paper, the spherical symmetry solution of the Einstein's equation of gravity is transformed into flat space-time to describe. The results show that the experiments to support the general theory of relativity can also be explained. But the theory has no any singularity in strong field. Besides, there are more experiments and astronomic observations can be rationally explained. Based on this result, a more rational theory of gravity can be established in the form of electromagnetic theory with the Lorentz invariability. In this way, the gravitational and electromagnetic interactions can be described in a consistent form. Similar to electromagnetic theory, this kind of theory of gravity is easy to quantization and normalization. The detail will be provided in next paper.

Section 4 The Revised Formulas of Newtonian Gravity Based on the Schwarzschild Solution of the Einstein's Equation of Gravitation fields

1. Impropriety of direct calculation and measurement in curved space-time

According to the Einstein's theory of gravity, the space-time of gravitational field is curved. But can we determine the curvature of space-time through the direct measurements of observers who are located in gravitational fields? The answer is no. This because that we should define standard rulers and clocks for the meaningful measurements. But standard rulers and clocks can not be defined in curved space-time, it only be done in flat space-time. If there are no the definitions of standard rulers and clocks, we can not establish the meaningful concepts of distances, angles and time intervals and so on with measurement significance. However, when we move the standard rulers and clocks defined in flat space-time into gravitational fields, the rulers and clocks would become "curved" synchronously so that we can not use them to measure space-time's curvature of gravitational fields. The measurement's results in curved space-time are meaningless in practices if they can not be compared with that in flat space-time. On the other hand, according to the strong principle of equivalence, we may introduce local inertial reference frames in gravitational fields in which standard rulers and clocks can be defined. But in local inertial reference frames, gravitational fields are considered to disappear. So even though we may define standard rulers and clocks in local inertial reference frames, they are useless. In fact, in the current calculations of gravitational problems based on general relativity, no standard rulers and clocks of the local inertial reference frames are used. We always calculate the problems directly in curved space-time.

On the other hand, the gravitational field of the earth is quite weak so that it can be considered to be approximately flat. In fact, all experimental verifications about the Einstein's theory of gravitation are completed on the earth. So we should transform the theoretical predictions of general relativity calculated in curved space-time into that in flat space-time, then comparing the results with the experiments carried on the earth. Only in this way, we can say that the Einstein's theory of gravitation is correct or wrong. At present, all calculations of concrete problems in the Einstein's theory of gravitation are carried out in the curved space-time. For example, the angles of the perihelion precession of the Mercury and the deviation of light in the solar gravitational field, the time intervals of delay effect of radar waves and so on, all of theses quantities are defined in curved space-time. Unfortunately, these calculation results in curved space-time are always compared directly with the experiments carried out on the earth without transforming them into the results of flat space-time at present. Therefore, it is improper to affirm that the Einstein's theory of gravity has been verified before we do them. Otherwise the judgment would be relaxed and trustless. This is a principle problem, but it is neglected completely at present.

The Einstein's theory of gravity based on curved space-time has become the mainstream of gravitational research. But there exist same insurmountable difficulties in it, just as the problems of renormalization and singularity, the definition of gravitational field's energy and so on. On the other hand, because the Einstein's equations of gravitational fields are nonlinear, the idea to establish the theory of gravity based on flat space-time is always attractive. From the 1940's, lots of persons tried to re-establish the gravitational theory in flat space-time [(14)]. These theories are equivalent with the Einstein's theory under the conditions of weak fields, but in general situations they are different. But at present, no experiments can prove that these theories are superior to the Einstein's theory. So in light of common viewpoint, the space-time of gravitational fields should be described by non-Euclidean geometry. The flat space-time is always regarded as the boundary condition far away from gravitational fields.

On the other hand, according to the non-Euclidean geometry, the metrics of curved space-time can not be transformed into that of flat space-time in general, otherwise the curved space would not be the real curved one. This conclusion seems to indicate that gravitational fields can not be described in flat space-time. However, this impossibility only means that we can not transform the whole metrics of gravitational fields from the non-Euclidean into the Euclidean. But we can always transform a curve described in curved space into that described in flat space. In fact, only by observing the object's motions in gravitational fields, we can comprehend the nature of gravitational field's space-time. According to the general theory of relativity, objects always move along the geodetic lines in gravitational fields. As long as we transform the geodetic lines defined in curved space-time into the curved lines or the motion equations defined in flat space-time, we can transform the theory of gravity described in curved space-time into that described in flat space-time. In this way, we can compare the theoretical predictions based on curved space-time with the experiments carried out in flat space-time. It is unnecessary for us to transform whole curved space-time into flat space-time.

2. The Schwarzschild solution described in flat space-time

Now let's discuss how to transform the Schwarzschild metric of the Einstein's equation of gravitational field into flat space-time to description. According to the general theory of relativity, the Schwarzschild metric of static mass distribution with spherical symmetry (external solution) is

$$ds^2 = c^2\left(1-\frac{\alpha}{r}\right)dt^2 - \left(1-\frac{\alpha}{r}\right)^{-1} dr^2 - r^2\left(\sin^2\theta d\varphi^2 + d\theta^2\right) \tag{4.1}$$

Here $\alpha = 2GM/c^2$. According to the familiar results in general relativity, let $\theta = \pi/2$ and substitute Eq.(4.1) into the equation of geodetic line, we get the integrals

$$c\left(1-\frac{\alpha}{r}\right)\frac{dt}{ds} = \varepsilon \qquad\qquad r^2\frac{d\varphi}{ds} = \frac{L}{c} \tag{4.2}$$

Here ε and L are constants. From above two formulas, we can eliminate linear element ds and get

$$r^2\left(1-\frac{\alpha}{r}\right)^{-1}\frac{d\varphi}{dt} = \frac{L}{\varepsilon} \tag{4.3}$$

Defining

$$d\tau = \left(1-\frac{\alpha}{r}\right)dt \tag{4.4}$$

Considering τ as the proper time, t as the coordinate time and let $\varepsilon = 1$, we have from Eq.(4.2)

$$ds = cd\tau \tag{4.5}$$

Thus, Eq.(4.4) becomes

$$r^2\frac{d\varphi}{d\tau} = L \tag{4.6}$$

Here L is the angel momentum of unit mass. Eq.(4.6) is just the conservation formula of angel momentum. Let's first discuss the motions of particles with static masses. By using Eq.(4.6), Eq.(4.1) can be written as

$$\left(1-\frac{\alpha}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - \left(1-\frac{\alpha}{r}\right)^{-1}\left(\frac{dr}{cd\tau}\right)^2 - r^2\left(\frac{d\varphi}{cd\tau}\right)^2 = 1 \tag{4.7}$$

From Eq.(4.5) and (4.7), we get

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{c^2\alpha}{r}\left(1-\frac{L^2}{\alpha c^2 r} + \frac{L^2}{c^2 r^2}\right) \tag{4.8}$$

By the differential about $d\tau$ in the formula above, we get

$$\frac{d^2r}{d\tau^2}-\frac{L^2}{r^3}=-\frac{c^2\alpha}{2r^2}\left(1+\frac{3L^2}{c^2r^2}\right) \tag{4.9}$$

It should be noted that all quantities in Eq.(4.9) are defined in the caved space-time. In order to describe the geodetic line equation in flat space-time, coordinate transformation is needed. Let r_0, φ_0 and t_0 represent the space-time coordinates of flat space-time, due to the invariability of the 4-diamention interval ds^2, we have

$$ds^2=c^2dt_0^2-dr_0^2-r_0^2d\varphi_0^2=c^2\left(1-\frac{\alpha}{r}\right)dt^2-\left(1-\frac{\alpha}{r}\right)^{-1}dr^2-r^2d\varphi^2 \tag{4.10}$$

It can be seen that the forms of the third items on the two sides of formula above are completely the same. The difference is only on the symbols. So we can take $r_0=r$, $\varphi_0=\varphi$ and then get the transformation between times t_0 and t

$$c^2dt_0^2=c^2\left(1-\frac{\alpha}{r}\right)dt^2+\left[1-\left(1-\frac{\alpha}{r}\right)^{-1}\right]dr^2 \tag{4.11}$$

On the other hand, by considering Eq.(4.4) and (4.8), we have

$$dr=c\left(1-\frac{\alpha}{r}\right)\sqrt{\frac{\alpha}{r}\left(1-\frac{L^2}{\alpha c^2r}+\frac{L^2}{c^2r^2}\right)}dt \tag{4.12}$$

Put the formula into Eq.(4.11), we have

$$dt_0=\sqrt{\left(1-\frac{\alpha}{r}\right)\left[1-\frac{\alpha^2}{r^2}\left(1-\frac{L^2}{\alpha c^2r}+\frac{L^2}{c^2r^2}\right)\right]}dt \tag{4.13}$$

Comparing it with Eq.(4.4), we get

$$d\tau=\left(1-\frac{\alpha}{r}\right)^{\frac{1}{2}}\left[1-\frac{\alpha^2}{r^2}\left(1-\frac{L^2}{\alpha c^2r}+\frac{L^2}{c^2r^2}\right)\right]^{-\frac{1}{2}}dt_0 \tag{4.14}$$

Because we have defined $r_0=r$, all quantities on the light side of formula above have been defined in flat space-time. On the other hand, in the classical Newtonian theory of gravity, the motion equations of unit mass in the plane polar coordinates are

$$\frac{d^2r}{dt^2}-r\left(\frac{d\varphi}{dt}\right)^2=-\frac{c^2\alpha}{2r^2} \tag{4.15}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\varphi}{dt}\right)=0 \qquad \text{or} \qquad r^2\frac{d\varphi}{dt}=L \tag{4.16}$$

Substituting Eq.(4.16) into Eq.(4.15), we obtain

$$\frac{d^2r}{dt^2}-\frac{L^2}{r^3}=-\frac{c^2\alpha}{2r^2} \tag{4.17}$$

Comparing Eq.(4.17) with Eq.(4.9), we know that besides the revised item on the light side of Eq.(4.9), as long as take $\tau \leftrightarrow t$, the forms of Eqs.(4.9) and (4.17) are completely the same. So we can write Eq.(4.9) in the vector form similar to the Newtonian gravitational theory

$$\frac{d^2\vec{r}}{d\tau^2} = -GM\left(1+\frac{3L^2}{c^2r^2}\right)\frac{\vec{r}}{r^3} \tag{4.18}$$

In the formula, all quantities in the formula have been defined in flat space-time. Let $u = 1/r$ and by considering Eq.(4.6), the formula above can be transformed into

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{L^2}\left(1+\frac{3L^2u^2}{c^2}\right) \tag{4.19}$$

This formula can be used to describe the perihelion precession of the Mercury. Now let's prove that the effect of special relativity has been taken into account in Eq.(4.18). From Eqs.(4.8), (4.10)----(4.14), we can obtain

$$V_r^2 = \left(\frac{dr}{dt_0}\right)^2 = \left(\frac{dr}{d\tau}\frac{d\tau}{dt_0}\right)^2 = \frac{c^2\alpha}{r}\left(1-\frac{\alpha}{r}\right)\left(1-\frac{L^2}{\alpha c^2 r}+\frac{L^2}{c^2r^2}\right)\left[1-\frac{\alpha^2}{r^2}\left(1-\frac{L^2}{\alpha c^2 r}+\frac{L^2}{c^2r^2}\right)\right]^{-1} \tag{4.20}$$

$$V_\varphi^2 = \left(r\frac{d\varphi}{dt_0}\right)^2 = \left(r\frac{d\varphi}{d\tau}\frac{d\tau}{dt_0}\right)^2 = \frac{L^2}{r^2}\left(1-\frac{\alpha}{r}\right)\left[1-\frac{\alpha^2}{r^2}\left(1-\frac{L^2}{\alpha c^2 r}+\frac{L^2}{c^2r^2}\right)\right]^{-1} \tag{4.21}$$

$$V^2 = V_r^2 + V_\varphi^2 = \frac{c^2\alpha}{r}\left(1-\frac{\alpha}{r}\right)\left(1+\frac{L^2}{c^2r^2}\right)\left[1-\frac{\alpha^2}{r^2}\left(1-\frac{L^2}{\alpha c^2 r}+\frac{L^2}{c^2r^2}\right)\right]^{-1} \tag{4.22}$$

$$1-\frac{V^2}{c^2} = \left(1-\frac{\alpha}{r}\right)\left[1-\frac{\alpha^2}{r^2}\left(1-\frac{L^2}{\alpha c^2 r}+\frac{L^2}{c^2r^2}\right)\right]^{-1} \tag{4.23}$$

Comparing with Eq.(4.14), we get

$$d\tau = \sqrt{1-\frac{V^2}{c^2}}\,dt_0 \tag{4.24}$$

This is just the formula of time delay in special relativity. At last, let $t_0 \to t$, Eq.(4.18) can be written as

$$\frac{d\vec{p}}{dt} = -GMm_0\left(1+\frac{3L^2}{c^2r^2}\right)\sqrt{1-\frac{V^2}{c^2}}\frac{\vec{r}}{r^3} \tag{4.25}$$

Here m_0 the static mass of a particle. The formula can be regarded as the revision of the Newtonian formula of gravity. In this way, Eq.(4.6) can be written as

$$r^2\frac{d\varphi}{d\tau} = \frac{r^2\dot{\varphi}}{\sqrt{1-V^2/c^2}} = L \tag{4.26}$$

So in the center gravitational fields, the classical angle momentum $m_0r^2\dot{\varphi}$ is not a constant again. It should be divided by a contraction factor of special relativity.

3. The motion of particle in gravitational field with spherical symmetry

The problem of energy conservation is discussed below. For simplification, we only discuss the situation that a particle moves along the radius vector direction with $L=0$. By considering Eq.(4.23) in this case, Eq.(4.25) becomes

$$\frac{d\vec{p}}{dt}=-\frac{GMm_0}{\sqrt{1+\alpha/r}}\frac{\vec{r}}{r^3} \tag{4.27}$$

By producing $d\vec{r}$ on the two sides of Eq.(4.27), the potential energy of gravitational field is

$$U(r)=-\int\vec{F}\cdot d\vec{r}=\int\frac{m_0c^2\alpha}{2\sqrt{1+\alpha/r}}\frac{\vec{r}}{r^3}\cdot d\vec{r}=-m_0c^2\sqrt{1+\frac{\alpha}{r}}+A_1 \tag{4.28}$$

Here A_1 is a constant. When $r\to\infty$, we have $U\to 0$ and get $A_1=m_0c^2$. The integral on the left side of Eq.(4.27) can be written as

$$T=\int\frac{d\vec{p}}{dt}\cdot d\vec{r}=\int\frac{d\vec{p}}{dt}\cdot\frac{d\vec{r}}{dt}dt=\int\vec{V}\cdot d\vec{p}=\vec{V}\cdot\vec{p}-\int\vec{p}\cdot d\vec{V}$$

$$=\frac{m_0V^2}{\sqrt{1-V^2/c^2}}+m_0c^2\sqrt{1-\frac{V^2}{c^2}}+A_2 \tag{4.29}$$

Here T is just the dynamic energy of particle. When $r\to\infty$, we have $V=0$ and get $A_2=-m_0c^2$. So the law of energy conservation of a particle in the gravitational field can be written as

$$T+U=\frac{m_0V^2}{\sqrt{1-V^2/c^2}}+m_0c^2\left(\sqrt{1-\frac{V^2}{c^2}}-1\right)-m_0c^2\left(\sqrt{1+\frac{\alpha}{r}}-1\right)=0 \tag{4.30}$$

When $\alpha/r<<1$, $V<<c$, from Eq.(4.30) we get the classical law of energy conservation in the Newtonian theory of gravity

$$\frac{m_0V^2}{2}-\frac{GMm_0}{r}=0 \tag{4.31}$$

For the situation with $L\neq 0$, we can also calculate the problem in the weak field with $\alpha/r<<1$. By remaining items with the orders up to r^{-2}, we have

$$U(r)=-\int\vec{F}\cdot d\vec{r}=\int\frac{m_0c^2\alpha}{2\sqrt{1+\alpha/r}}\left(1+\frac{3L^2}{c^2r^2}\right)\frac{\vec{r}}{r^3}\cdot d\vec{r}$$

$$=-m_0c^2\left\{\sqrt{1+\frac{\alpha}{r}}-\frac{3L^2}{c^2\alpha^2}\left(1+\frac{\alpha}{r}\right)\left[\frac{1}{6}\left(1+\frac{\alpha}{r}\right)^2-\frac{2}{3}\left(1+\frac{\alpha}{r}\right)^{1/2}+1\right]\right\}+A_1 \tag{4.32}$$

So the law of energy conservation is

$$E=\frac{m_0V^2}{\sqrt{1-V^2/c^2}}+m_0c^2\left\{\sqrt{1-\frac{V^2}{c^2}}-\sqrt{1+\frac{\alpha}{r}}\right.$$

$$\left.+\frac{3L^2}{c^2\alpha^2}\left(1+\frac{\alpha}{r}\right)\left[\frac{1}{6}\left(1+\frac{\alpha}{r}\right)^2-\frac{2}{3}\left(1+\frac{\alpha}{r}\right)^{1/2}+1\right]\right\} \tag{4.33}$$

Here E is a constant.

Let's now discuss the motion of an experimental particle along the direction of radius in the curved Schwarzschild coordinate. After that, we discuss the problem in flat space-time. From Eq.(4.4) and (4.8) in curved space-time, when $L=0$, we get particle's speed

$$V=\frac{dr}{dt}=\pm c\sqrt{\frac{\alpha}{r}}\left(1-\frac{\alpha}{r}\right) \tag{4.34}$$

Within the region $r>\alpha$, when the particle moves along the positive direction of radius vector, the formula takes positive sign. When the particle moves along the negative direction of radius vector, the formula takes negative sign. Within the region $r<\alpha$, when the particle moves along the positive direction of radius vector, the formula takes negative sign and when the particle moves along the negative direction of radius vector, the formula takes positive sign. It is obvious that when $r\to\infty$, we have $V=0$. When $r=\alpha$, i.e., the particle reaches the event horizon, we also have $V=0$. Within the region $\sqrt{\alpha/r}(1-\alpha/r)<1$, particle's speed is less than light's speed in vacuum. Within the region $\sqrt{\alpha/r}(1-\alpha/r)>1$, the particle's speed surpasses light's speed in vacuum. When $r\to 0$, we have $V\to\infty$. From the formula above, particle's acceleration is

$$a=\frac{dV}{dt}=-\frac{1}{2}\frac{c^2\alpha}{r^2}\left(1-\frac{\alpha}{r}\right)\left(1-\frac{3\alpha}{r}\right) \tag{4.35}$$

At the points $r=3\alpha$ and $r=\alpha$, particle's accelerations are zero. Within the region $r>3\alpha$, we have $a<0$. It means that particle is acted by gravitation so its speed is increased. Within the region $\alpha<r<3\alpha$, we have $a>0$. The particle is acted by repulsion force and its speed is decreased. (It is inconceivable in this case that gravitational force becomes repulsion force.). When $r=\alpha$, we have $V=a=0$. In this case, particle is at rest on the event horizon without the action of force. When $r<\alpha$, we have $a<0$, particle is acted by gravitation and moves towards the center of gravitational field. When $r\to 0$, we also have $a\to\infty$.

Let's consider the integral of Eq.(4.35). Because there exist singularity at point $r=\alpha$, the integrals should be taken individually in different regions. Within the region $r>\alpha$ with the initial condition $r=r_0$ when $t=0$, the integral of Eq.(4.34) is

$$ct=\pm\frac{1}{\sqrt{\alpha}}\left[\frac{2}{3}\left(r^{3/2}-r_0^{3/2}\right)+2\alpha\left(\sqrt{r}-\sqrt{r_0}\right)+\alpha^{3/2}\ln\frac{(\sqrt{r}-\sqrt{\alpha})(\sqrt{r_0}+\sqrt{\alpha})}{(\sqrt{r}+\sqrt{\alpha})(\sqrt{r_0}-\sqrt{\alpha})}\right] \tag{4.36}$$

Suppose the particle falls down in gravitational field, we take negative sign in the formula above. When $r\to\alpha$, we heve $t\to\infty$. It means that the particle needs an infinite time to reach the event horizon. Then suppose that the particle's initial position is at $r_0\to\alpha$. We take positive sign so that the particle moves up apart from the event horizon. Suppose that the particle reaches $r=\alpha+\Delta$ point at a certain time, Δ may be a very small but limited value. Because the third item of Eq.(4.36) would become infinite when $r_0\to\alpha$, we have $t\to\infty$ in this case. It means that the particle on the event horizon can not move up apart from the event horizon actually.

Then we discuss particle's motion within the event horizon with $r<\alpha$. Suppose that the particle is at point $r=r_0$ when $t=0$, the integral of Eq.(4.34) is

$$ct=\pm\frac{1}{\sqrt{\alpha}}\left[\frac{2}{3}\left(r^{3/2}-r_0^{3/2}\right)+2\alpha\left(\sqrt{r}-\sqrt{r_0}\right)+\alpha^{3/2}\ln\frac{(\sqrt{\alpha}-\sqrt{r})(\sqrt{\alpha}+\sqrt{r_0})}{(\sqrt{\alpha}+\sqrt{r})(\sqrt{\alpha}-\sqrt{r_0})}\right] \tag{4.37}$$

We take $r_0=0$ at first. According to Eq.(4.34), the initial speed of particle would be infinite. Suppose that particle moves up, we take negative sign in the formula above and get

$$ct = -\frac{1}{\sqrt{\alpha}}\left(\frac{2}{3}r^{3/2} + 2\alpha\sqrt{r} + \alpha^{3/2}\ln\frac{\sqrt{\alpha}-\sqrt{r}}{\sqrt{\alpha}+\sqrt{r}}\right) \tag{4.38}$$

It is easy to verify that the time is a positive number within the region $r < \alpha$ in the formula above. When particle reaches the event horizon, we have $t \to \infty$. Then suppose that the initial position of particle is on the event horizon with $r_0 \to \alpha$. By the action of gravitation, particle moves down towards the center of gravitational field. We take positive sign in Eq.(4.37). Suppose that the particle reaches $r = \alpha - \Delta$ point at a certain time, Δ may be a very small but limited value. Because the third item of Eq.(4.37) would become infinite when $r_0 \to \alpha$, we have $t \to \infty$ in this case. It means that the particle on the event horizon can not move down continuously and then collapse at the center singularity of gravitational field after it reaches the event horizon. The results indicate that the event horizon is actually an attractive plane for moving particles. The particles could not leave the event horizon along both up and down directions after they had reached the event horizon. But this result is neglected in the current theory of black holes. According to the current understanding, any particles would move towards to the renter of gravitational field so that they would collapse at the center singularity at last.

It is obvious that there are some things irrational in the processes of particle's motions in the gravitational fields, besides the singularity of the event horizon. For example, some time the particle would be accelerated and some time it would be decelerated outside the event horizon. Especially particles would move in the speeds surpassing light's speed in vacuum, even move in an infinite speed. In the current theory, those problems are attributed to the improper selections of coordinates. In order to eliminate those defects, some coordinate systems just as the Eddington's and the Kruskal's coordinate systems (15) are introduced. In new coordinate systems, though the singularities on the event horizons may be eliminated, they can not yet be eliminated at the original point $r = 0$. Hawking etc. even proved that it was impossible to eliminate all singularities in the general theory of relativity (16).

However, as shown in former paper, it should be emphasized again that the arbitrary transformation in four dimension space-time is impossible for gravitational problems. This kind of transformations would introduce arbitrary inertial forces, and the inertial forces are considered to equivalent to arbitrary gravitational fields. So after the transformations, the new gravitational fields are not equal to original ones. In the current theories of black holes, in order to eliminate singularity on the event horizon, the freely falling Novikow or Lemaitre coordinates are introduced. Because there are no the event horizons, observers can enter the event horizons without any felling. After that, according to the current understanding, they would be attracted into the center of black holes and be torn into the pieces by so-called tide forces at last. But in the Schwarzschild coordinate, the observers would stop on the event horizon forever. However, the observer's life and death are absolute events, what are observer's fates? In principle, we can find infinite coordinate systems in which the singularities on the event horizons can be eliminated. But none of theses metrics are with spherical symmetry except the Schwarzschild coordinate. How can they represent the gravitational field with spherical symmetry? On the other hand, in the curved coordinate systems, no matter in the Novikow or Lemaitrewe coordinate systems, we can not define stander rulers and clocks. In theses curved coordinate systems, the speeds of clocks located at different places are not the same so that the measurements of time intervals are meaningless. For example, we say that an observer freely falling down the gravitational field would spend infinite time to reach the event horizon. Because the clock's speed in the moving reference frame changes continuously, what is the real meaning of infinite time? Because only in the flat space-time we can define stander rulers and clocks, only in the flat reference frames outside the gravitational fields, the calculations and measurements for objects moving in the gravitational fields are meaningful. So the problems of gravitation should be transformed into the flat space-time for discussion.

Now let's discuss the motion of a particle in the gravitational field from the angle of observers in flat space-time outside gravitational field. Suppose that a particle falls freely along the radium direction of gravitational field, its velocity and acceleration are individually

$$V = \frac{dr}{dt} = -c\sqrt{\frac{\alpha}{r}}\left(1 + \frac{\alpha}{r}\right)^{-1/2} \tag{4.39}$$

$$a = -\frac{1}{2}\frac{c^2\alpha}{r^2}\left(1 + \frac{\alpha}{r}\right)^{-2} \tag{4.40}$$

It is known that when $r \to \infty$, we have $V = 0$ and $a = 0$. Suppose when $t = 0$ the particle is at point $r = r_0$, by taking the integral of Eq.(4.39), we get

$$ct = \frac{2}{3\sqrt{\alpha}}\left[(r_0 + \alpha)^{3/2} - (r + \alpha)^{3/2}\right] \tag{4.41}$$

It is obvious that every thing is normal within the region $r > 0$. The particle is monotonously accelerated by gravitation. There is no any singularity in the whole space-time. When particle arrives at the original point $r = 0$, we have

$$V = -\lim_{x\to\infty}\frac{c\sqrt{\alpha/r}}{\sqrt{1+\alpha/r}} \to -c \qquad\qquad a = -\lim_{x\to\infty}\frac{c^2x^2}{2\alpha(x+1)^2} \to -\frac{c^2}{2\alpha} \tag{4.42}$$

$$F = -\lim_{x\to\infty}\frac{c^2x^2}{2\alpha(x+1)} \to -\lim_{x\to\infty}\frac{c^2x}{2\alpha} \to -\infty \tag{4.43}$$

It shows that the speed of particle tends to light's speed in vacuum. Acceleration is also finite. So within the region $0 < r \leq \infty$, the motion of particles with static masses are continuous. Only at point $r = 0$, the force acted on particles are infinite. But this infinite also appears in the Newtonian theory of gravitation, having nothing to do with space-time singularity. When a particle moves along the positive direction of radius vector, as long as its velocity satisfies Eq.(4.39), the particle would escape gravitational field and has a speed $V = 0$ when it reach the place $r \to \infty$. That is to say, after the Schwarzschild solution is transformed into flat space-time to describe, for the motion of particles with static mass, all space-time singularities disappear. So it is obvious that singularities appearing in the Einstein's theory of gravity are actually caused by the descriptive method in curved space-time. As long as we describe the problems og gravitation in flat space-time, all singularities are canceled. The gravitational field itself has no singularities. In real and physical world, singularity is not allowed to exist.

4. Photon's motions in gravitational field with spherical symmetry

The motion equation of photon in flat space-time is discussed as follows. For photons, we have $ds = 0$. In this case, we take $d\tau$ as parameter to describe the equation of geodetic line. By solving the Einstein's equation of gravity with spherical symmetry, we obtain

$$\left(1 - \frac{\alpha}{r}\right)\frac{dt}{d\tau} = \varepsilon \qquad\qquad r^2\frac{d\varphi}{d\tau} = L \tag{4.44}$$

Let $\varepsilon = 1$ similarly, we have

$$d\tau = \left(1 - \frac{\alpha}{r}\right) dt \tag{4.45}$$

Because of

$$ds^2 = c^2\left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 d\varphi^2 = 0 \tag{4.46}$$

the formula above can be written as

$$c^2\left(1 - \frac{\alpha}{r}\right)\left(\frac{dt}{d\tau}\right)^2 - \left(1 - \frac{\alpha}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 - \left(r\frac{d\varphi}{d\tau}\right)^2 = 0 \tag{4.47}$$

From the formula, we obtain

$$\left(\frac{dr}{d\tau}\right)^2 = c^2\left[1 - \left(1 - \frac{\alpha}{r}\right)\frac{L^2}{c^2 r^2}\right] \tag{4.48}$$

By taking the differential of Eq.(4.48) about $d\tau$, we get

$$\frac{d^2 r}{d\tau^2} - \frac{L^2}{r^3} = -\frac{3\alpha L^2}{2r^4} \tag{4.49}$$

By using Eqs.(4.45) and (4.47), we have

$$\left(\frac{dr}{dt}\right)^2 = c^2\left(1 - \frac{\alpha}{r}\right)^2\left[1 - \left(1 - \frac{\alpha}{r}\right)\frac{L^2}{c^2 r^2}\right] \tag{4.50}$$

Suppose that photon's speed in the gravitational field is V, from Eqs.(4.41), (4.42) and (4.47), we get

$$V = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\varphi}{dt}\right)^2} = c\left(1 - \frac{\alpha}{r}\right)\sqrt{1 + \frac{\alpha L^2}{c^2 r^3}} \tag{4.51}$$

It is obvious that the speed of light in the gravitational field would change with $V \neq c$ in general. Then suppose that photon moves along the radius vector's direction with $L = 0$. The velocity of photon is

$$V = \frac{dr}{dt} = \pm c\left(1 - \frac{\alpha}{r}\right) \tag{4.52}$$

When $r \to \infty$ we have $V = 0$. When $r = \alpha$ we also have $V = 0$, i.e., particle's speed would be zero when it reaches the event horizon. Within the region $\alpha / r > 2$, we have $V > c$. When $r \to 0$ we have $V \to \infty$, i.e., particle's speed becomes infinite when particle arrive at the center point of gravitational field. From the formula above, we can obtain particle's acceleration

$$a = \frac{dV}{dt} = \frac{c^2\alpha}{r^2}\left(1 - \frac{\alpha}{r}\right) \tag{4.53}$$

So within the region $r > \alpha$, particle's acceleration is positive with $a > 0$. It indicates that particle is acted by repulsion force. When a particle falls down the gravitational field freely, its speed is decreased.

When it moves up apart from the field, its speed is increased. Similarly, space-time has singularity at point $r = \alpha$, so the integral should also be carried out in the different regions. When particle falls down the gravitational field, the negative sign is taken in Eq.(4.52). Within the region $r > \alpha$, let $r = r_0$ when $t = 0$, the integral of Eq.(4.52) is

$$ct = r_0 - r - \alpha \ln \frac{r - \alpha}{r_0 - \alpha} \tag{4.54}$$

It is known from Eq.(4.52) and (4.53) that photon's speed and acceleration are zero at point $r = \alpha$, so photons would stop at the event horizon. But from Eq.(4.54), it needs a infinite time for a photon to reach the event horizon. In this case, photons can not yet leave the event horizon, i.e., there exist so-called black holes. Within the event horizon with $r < \alpha$, particle's acceleration $a < 0$, so photon is acted by gravitation and its speed is increased. Suppose we have $r_0 \to \alpha$ when $t = 0$ and photon moves towards the center of gravitational field. The integral of Eq.(4.52) is

$$ct = r - r_0 + \alpha \ln \frac{\alpha - r}{\alpha - r_0} \tag{4.55}$$

Suppose the particle reaches $r = \alpha - \Delta$ point at a certain time, Δ may be a very small but limited value. Because the second item of Eq.(4.55) would become infinite when $r_0 \to \alpha$, we have $t \to \infty$ in this case. It means that the particle on the event horizon can not move up apart from the event horizon. So the particles could not leave the event horizon along both up and down directions after they had reached the event horizon. The event horizon is also an attractive plane for moving photons.

Then let's discuss how to describe photon's motion in flat space-time. For photon's motion, the metric in the flat space-time can be written as

$$ds^2 = c^2 dt_0^2 - dr_0^2 - r_0^2 d\varphi_0^2 = 0 \tag{4.56}$$

From Eqs.(4.47) and (4.54), we have

$$V_0^2 dt_0^2 - dr_0^2 - r_0^2 d\varphi = c^2\left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2 d\varphi^2 \tag{4.57}$$

Similarly, the forms of the third items on the two sides of above formula are the same completely, we can also let $r_0 = r$, $\varphi_0 = \varphi$ and get

$$c^2 dt_0^2 = c^2\left(1 - \frac{\alpha}{r}\right)dt^2 + \left[1 - \left(1 - \frac{\alpha}{r}\right)^{-1}\right]dr^2 \tag{4.58}$$

By using Eq.(4.50), we obtain

$$c^2 dt_0^2 = c^2\left(1 - \frac{\alpha}{r}\right)^2\left(1 + \frac{\alpha L^2}{c^2 r^3}\right)dt^2 \tag{4.59}$$

or

$$dt_0 = \left(1 - \frac{\alpha}{r}\right)\sqrt{1 + \frac{\alpha L^2}{c^2 r^3}}\,dt \tag{4.60}$$

$$d\tau = \frac{dt_0}{\sqrt{1+\alpha L^2/\left(c^2 r^3\right)}} \tag{4.61}$$

Similarly, the motion equation of photons in the gravitational field with spherical symmetry can be written in the form of vector in flat space-time

$$\frac{d^2\vec{r}}{d\tau^2} = -\frac{3\alpha L^2 \vec{r}}{2r^5} \tag{4.62}$$

Let $u = 1/r$, the formula above can be transformed into

$$\frac{d^2 u}{d\varphi^2} + u = \frac{3GM}{c^2} u^2 \tag{4.63}$$

The formula can be used to describe the deviation of light in the solar gravitational field. On the other hand, according to Eq.(4.50) and by taking approximation in weak field, we have

$$cdt = \frac{\left(1+\alpha/r\right)\left[1-\alpha L^2/\left(c^2 r^3\right)\right]}{\sqrt{1-L^2/\left(c^2 r^2\right)}} dr \tag{4.64}$$

From the formula, the delay experiment of radar wave can be explained well [3]. But if using Eqs.(4.58) and (4.59) and taking approximation of weak field, we have (let $t_0 \to t$ in the formula so that r and t have been the coordinates of flat space-time.)

$$cdt = \frac{\sqrt{1+\alpha L^2/\left(c^2 r^3\right)}}{\sqrt{1-L^2/\left(c^2 r^2\right)+\alpha L^2/\left(c^2 r^3\right)}} dr \approx \frac{1-\alpha L^4/\left(4c^4 r^5\right)}{\sqrt{1-L^2/\left(c^2 r^2\right)}} dr \tag{4.65}$$

It is obvious that if the formula above is used to calculate, the correct conclusion can not be reached. This result means that when the Schwarzschild metric is transformed into flat space-time, we can not describe correctly the delay effect of radar wave in the sun gravitational field.

As we discuss before, in order to compare the theoretical predictions of curved space-time with the experiments carried out in the flat space-time of the earth, we have to transform the descriptions of curved space-time into that of flat space-time. Otherwise theoretical prediction is meaningless. Because the current calculation on the delay effect of radar wave is based on curved space-time, the result can not coincide with experiments carried out in the flat space-time of the earth. So it is improper to say that the delay effect of radar wave has been explained well in the Einstein's theory of gravitation, though it is effect for the perihelion precession of the Mercury and the deviation of light in the solar gravitational field. This result is easy to understand because the first two experiments are only relative to the measurements of space distances, but the third experiment is relative to the measurement of time interval. In the discussion above, when the Schwarzschild solution is transformed into the fat space-time to describe, we let the space coordinates to be equal to each other in the curved and flat reference frames. But the definitions of times are different in both reference frames.

At present we use Eq.(4.62) to describe photon's motion in the gravitational field with spherical symmetry and compare the theoretical predictions directly with the experiments and astronomical observations carried out on the earth of which space-time is nearly flat. This result hints us actually that all quantities appearing in Eq.(4.62) have been considered as that defined in flat space-time. In the other words, the results tell us that we can use Eq.(4.62) directly to describe the deviation of light and the delay effect of radar wave in gravitational field . It is unnecessary for us to transform the formula into flat reference frame again. Therefore, we can take Eq.(4.62) directly as a basic equation to describe photon's motion in gravitational field and establish the theory of gravitation in flat reference frame. Because we suppose that Eq. (4.62) is defined in flat space-time and the curvature of flat space-time is always zero, there is no any space-time singularity problem again for the theory. So we can say that it is just by reconstructing the Einstein's theory, we establish the theory of gravitation in flat space-time.

5. Gravitational red shift and the essence of quasar's red shift

It should be indicated that Eq.(4.62) can not be considered as the dynamic equation of photon in gravitational field. According to the formula, we can write photon's momentum and force as formally

$$\vec{p} = m_0 \frac{\vec{V}}{1-\alpha/r} \qquad \vec{F} = \frac{d\vec{p}}{dt_0} = -\left(1-\frac{\alpha}{r}\right)\frac{3\alpha L^2 \vec{r}}{2r^5} \tag{4.66}$$

When photon moves along the radius vector direction with $L=0$, we have $\vec{F}=0$. It seems that photon is not acted by force. But according to Eq.(4.69), photon has acceleration, so it should be acted by force in this case. Therefore, Eq.(4.62) is not the dynamic equation of photon. On the other hand, by considering (4.66), we can write Eq.(4.62) as

$$\frac{d^2\vec{r}}{dt^2} = -\left(1-\frac{\alpha}{r}\right)^2 \frac{3\alpha L^2 \vec{r}}{2r^5} + \left(1-\frac{\alpha}{r}\right)\frac{\alpha V_r \vec{V}}{r^2} \tag{4.67}$$

Because of $\vec{V} = \vec{V}_r + \vec{V}_\varphi$, the formula means that when photon moves in gravitational field, it would be acted by a force in the $\vec{e}_\varphi$ direction which is vertical to the $\vec{e}_r$ direction. By using Eq.(4.67), we can write the formula above as

$$\frac{d^2\vec{r}}{dt^2} = \left(1-\frac{2L^2}{c^2r^2}-\frac{\alpha L^2}{c^2r^3}\right)\left(1-\frac{\alpha}{r}\right)^3 \frac{2GM\vec{e}_r}{r^2} \pm \sqrt{1-\frac{L^2}{c^2r^2}+\frac{\alpha L^2}{c^2r^3}}\left(1-\frac{\alpha}{r}\right)^3 \frac{2GML\vec{e}_\varphi}{cr^3} \tag{4.68}$$

The formula can only be regarded as the motion equation of classical particle without considering the inertial mass and gravitational mass of photon which would change with speed. So it is only a formula to represent photon's acceleration, instead of dynamic equation. Though for classical particles, both are the same. But as shown below, we can obtain the proper dynamic equation of photon based on it.

Let's deduce photon's motion equation under the condition $L=0$ at first. Suppose that there is a fictitious particle corresponding to photon, of which velocity $\vec{V}'$ is equal to the difference between photon's velocity $\vec{c}$ in vacuum and the velocity $\vec{V}$ in gravitational field with $\vec{V}' = \vec{c} - \vec{V}$. When photon's velocity $V=c$ we have $V'=0$. When photon falls down in gravitational field with speed $V<c$, we have $V'>0$. When $V=0$, we have $V'=c$. So similar to general particle with static mass, the fictitious particle is acted by gravitation instead of repulse force. Suppose the fictitious particle's mass is the same as the photon's equivalent static mass m_0, according to Eq.(4.18), the dynamic equation of fictitious particle can be written as

$$\frac{d\vec{p}'}{dt} = -GMm_0\sqrt{1-\frac{(c-V)^2}{c^2}}\frac{\vec{r}}{r^3} \tag{4.69}$$

On the other hand, we define the momentum of moving photon in gravitational field as

$$\vec{p} = \frac{m_0\vec{V}}{R(V)} \tag{4.70}$$

Here R is an unknown function. Let $\vec{p}' \to m_0\vec{c} - \vec{p}$ again and put it into Eq.(4.69), the dynamic equation of photon would be

$$\frac{d\vec{p}}{dt} = \sqrt{\frac{2cV - V^2}{c^2}}\frac{GMm_0\vec{r}}{r^3} \tag{4.71}$$

In general situations when $L \neq 0$, corresponding to Eq.(4.67) or (4.68), we have to consider the existence of force on the $\vec{e}_\varphi$ direction. So the general form of photon's dynamic equation in the gravitational field with spherical symmetry can be written as at last

$$\frac{d\vec{p}}{dt} = \sqrt{\frac{2cV - V^2}{c^2}}\left(1 + \frac{3L^2}{c^2r^2}\right)\frac{GMm_0\vec{r}}{r^3} + \vec{F}_\varphi = \vec{F}_r + \vec{F}_\varphi = \vec{F} \tag{4.72}$$

In fact, it is unnecessary for as to introduce fictitious particle. We can suppose directly that the dynamic equation of photon satisfy the formula above, as long as the calculation results coincide with practical situations. Then let's determine the forms of functions $\vec{F}_\varphi$ and R. From Eq.(4.70) we have

$$\frac{d\vec{p}}{dt} = \frac{m_0}{R}\frac{d^2\vec{r}}{dt^2} + m_0\vec{V}\frac{d}{dt}\frac{1}{R} = \vec{F} \tag{4.73}$$

So the acceleration of photon is

$$\frac{d^2\vec{r}}{dt^2} = R\left(\frac{\vec{F}}{m_0} - \vec{V}\frac{d}{dt}\frac{1}{R}\right) \tag{4.74}$$

Comparing Eq.(4.74) with Eq. (4.67), we get

$$R\left(\frac{\vec{F}}{m} - \vec{V}\frac{d}{dt}\frac{1}{R}\right) = -\left(1 - \frac{\alpha}{r}\right)^2\frac{3\alpha L^2\vec{r}}{2r^5} + \left(1 - \frac{\alpha}{r}\right)\frac{\alpha V_r\vec{V}}{r^2} \tag{4.75}$$

By decomposing the formula in the $\vec{e}_r$ and $\vec{e}_\varphi$ directions, we get two formulas

$$R\left(\frac{F_r}{m_0} - V_r\frac{d}{dt}\frac{1}{R}\right) = -\left(1 - \frac{\alpha}{r}\right)^2\frac{3\alpha L^2}{2r^4} + \left(1 - \frac{\alpha}{r}\right)\frac{\alpha V_r^2}{r^2} \tag{4.76}$$

$$R\left(\frac{F_\varphi}{m_0} - V_\varphi\frac{d}{dt}\frac{1}{R}\right) = \left(1 - \frac{\alpha}{r}\right)\frac{\alpha V_r V_\varphi}{r^2} \tag{4.77}$$

Eq.(4.76) can be rewritten as

$$\frac{dR}{dt} + P(r)R = Q(r)R^2 \tag{4.78}$$

$$P(r) = \left(1 - \frac{\alpha}{r}\right)^2\frac{3\alpha L^2}{2V_r^2r^4} - \left(1 - \frac{\alpha}{r}\right)\frac{\alpha}{r^2} \qquad\qquad Q(r) = -\frac{F_r}{m_0V_r^2} \tag{4.79}$$

After we complete the integral of Eq.(4.78) and obtain the concrete form of R function, putting it into Eq.(4.77), we can know the form of F_φ. We can also write Eq.(4.77) as

$$-R\frac{d}{dt}\frac{1}{R} = \left(1 - \frac{\alpha}{r}\right)\frac{\alpha V_r}{r^2} - R\frac{F_\varphi}{m_0V_\varphi} \tag{4.80}$$

Put it into Eq.(4.76) and get

$$\vec{F}_\varphi = \left[F_r + \left(1 - \frac{\alpha}{r}\right)^2 \frac{3m_0\alpha L^2}{2Rr^4} \right] \frac{\vec{V}_\varphi}{V_r} = \left(F_r + \frac{3GMm_0 V_r^2 L^2}{Rc^4 r^4} \right) \frac{\vec{V}_\varphi}{V_r}$$

$$= \sqrt{\frac{2cV - V^2}{c^2}} \frac{GMm_0 \vec{V}_\varphi}{r^2 V_r} \left\{ 1 + \frac{3L^2}{c^2 r^2} \left(1 + \frac{V_r^2}{Rc\sqrt{2cV - V^2}} \right) \right\} \tag{4.81}$$

Here $V_\varphi / V_r = L\sqrt{1 - L^2/c^2r^2 + \alpha L^2/c^2r^3}\,/(cr)$ and R is determined by Eq.(4.78). The results are completely the same when we calculate photon's velocity and acceleration based on Eq.(4.72) and (4.67). So Eq.(4.72) can also be used to describe the three experiments to support general relativity. We call $\vec{F}_r$ as the longitudinal force and $\vec{F}_\varphi$ as the transverse force. Photons are acted by both longitudinal and transverse forces when they move in static gravitational fields with spherical symmetry. This is different from the other particles with static masses. The particles with static masses are only acted by the longitudinal force in this case.

In general situations, the integral of Eq.(4.78) is difficult. But we can do it when photon moves in the direction of radium with $L = 0$. In this case, we have $V_r = V = c(1 - \alpha / r)$ and get

$$\sqrt{\frac{2cV - V^2}{c^2}} = \sqrt{1 - \frac{\alpha^2}{r^2}} \qquad\qquad F_r = \sqrt{1 - \frac{\alpha^2}{r^2}} \frac{m_0 c^2 \alpha}{2r^2} \tag{4.82}$$

Substitute it into Eq.(4.72) and let $\alpha / r = x$, we get

$$\frac{1}{R} = exp\left(\int Pdr\right)\left[-\int Q\, exp\left(-\int Pdr\right)dr\right] = -\frac{(1 + \alpha / r)^2}{4(1 - \alpha / r)} = -\frac{(2c - V)^2}{4cV} \tag{4.83}$$

So when photon moves along the radius direction in the gravitational field with spherical symmetry, the momentum and the force acted on it are individually

$$\vec{P} = \frac{m_0 \vec{V}}{R} = -\frac{m_0 (2c - V)^2 \vec{V}}{4cV} \tag{4.84}$$

The corresponding moving mass of photon should be defined as

$$m = m_0 \frac{(2c - V)^2}{4cV} \tag{4.85}$$

The force acted on photon and acceleration can be written as

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{m_0 (2c - V)}{2c} \frac{d^2 \vec{r}}{dt^2} = \frac{\sqrt{2cV - V^2}}{c} \frac{GMm_0 \vec{r}}{r^3} \tag{4.86}$$

$$\frac{d^2 \vec{r}}{dt^2} = \sqrt{\frac{V}{2c - V}} \frac{2GM\vec{r}}{r^3} \tag{4.87}$$

It shows that the force acted on photon is repulse one, instead of gravitation. This is easy to understand. If photon is acted by gravitation, its speed would increase generally when it moves in gravitational field so that its speed would surpass light's speed in vacuum. However, this is impossible in general.

The problem of red shift of spectrum in gravitational field is discussed bellows. We only consider photon's motion along the radius vector direction with $L = 0$. Similar to Eq.(4.29), by multiplying $d\vec{r}$ on the left side of Eq.(4.72) and considering Eq.(4.82), we can obtain the integral

$$T=-m_0c^2\left(4-\frac{3V^2}{c^2}+\frac{V^3}{c^3}\right)+A_3 \tag{4.88}$$

T is actually the momentum of photon in gravitational field. When $r=r_0\to\infty$, we have $V=c$. So we get $T=T_0=-2m_0c^2+A_3$. Because free photon's dynamic energy is $T_0=m_0c^2=h\nu_0$, ν_0 is the frequency of free photon when $r\to\infty$. We can take $A_3=3m_0c^2$ and have

$$T=-m_0c^2\left(1-\frac{3V^2}{c^2}+\frac{V^3}{c^3}\right) \tag{4.89}$$

By multiplying $d\vec{r}$ on the light side of Eq.(4.72) and considering Eqs.(4.72) and (4.82), and taking the integral, we get photon's potential energy in the gravitational field with spherical symmetry under the condition $U(r\to\infty)$

$$U(r)=-\int\vec{F}_r\cdot d\vec{r}=-\int\sqrt{1-\frac{\alpha^2}{r^2}}\frac{m_0c^2\alpha}{2r^2}dr=\frac{m_0c^2}{4}\left(\frac{\alpha}{r}\sqrt{1-\frac{\alpha^2}{r^2}}+arcsin\frac{\alpha}{r}\right) \tag{4.90}$$

In this case, the energy conservation formula of photon in the gravitational field is

$$T+U=-m_0c^2\left(1-\frac{3V^2}{c^2}+\frac{V^3}{c^3}\right)+\left(\frac{\alpha}{r}\sqrt{1-\frac{\alpha^2}{r^2}}+arcsin\frac{\alpha}{r}\right)=h\nu_0 \tag{4.91}$$

In the weak field with $\alpha/r<<1$, by developing the formula into the Taylor's series, we have

$$U(r)=\frac{m_0c^2}{4}\frac{2\alpha}{r}=\frac{GMm_0}{r} \tag{4.92}$$

The photon's potential is the same as classical particle but take positive value. It means that photon is acted by repulsion force in gravitational field. When $\alpha/r=1$, we have $U(r)=m_0c^2\pi/8$. When $\alpha/r>1$, potential becomes imaginary number. This is meaningless, so it indicates that photon can not enter the region $r<\alpha$. In other words, if a star's radius $r<GM/c^2$, no photon can moves in the direction of radium within the region $r<\alpha$, though it is allowed for photons to move around the center of star with angle momentum $L\neq 0$. This kind of stars can not radiate so they can be regarded as black holes. But this kind of black holes has no space-time singularities. Space-time is normal in them. The event horizon is actually a potential base with highness $U(r=\alpha)=m_0c^2\pi/8$ which photon can not pass through. Photon's speed becomes zero when it reaches the potential.

As mentioned before that the Einstein's theory of gravity can not be a universal one. The effectiveness of the Schwarzschild solution can only be considered as a haphazard but excellent coincidence in weak field. We should reestablish gravitational theory in flat space-time based on the spherical symmetry solution the Einstein's equation. In fact, we can consider Eqs.(4.25) and (4.72) as the basic dynamic equations of particles moving in the gravitational fields caused by other static particles. Then by means of the method of force's superposition, we can construct gravitational interactions among the bodies with any different forms. No any other gravitational equations in curved space-time are needed again. In this way, we can establish the general theory of gravity in the form of electromagnetic interaction. The detail will be provided in the next section.

Section 5 Gravitational Theory Established in Flat Space-time

1. The gravitational theory between objects with static masses

As we shown before that it is improper to consider the Einstein's theory of gravity as the foundational interaction theory of gravity. The real value of the Einstein's theory is to provide the Schwarzschild metric of static gravitational field with spherical symmetry. It is useful to describe object's motions in the weak field, though it may be an accidental coincident. The later theory should be consistent with it. After the metric is transformed into flat space-time for discussion, the dynamic equation of gravitational interaction between two particles can be obtained. By considering the similarity between classical electromagnetic and gravitational theories as well as by introducing some proper hypothesis, we can establish a more rational gravitational theory with the Lorentz invariability. The descriptions of electromagnetic and gravitational interactions can also become consistent.

In the first article, we prove that the absolutely resting reference frame should exist. In the absolutely resting reference frame, an object's mass would be smallest. So we should establish gravitational theory in the absolutely resting reference frame firstly. After that, we transform the theory into other inertial reference frames for discussion. Let m_{i0} represent the static mass of a particle in the absolutely resting reference frame, $\vec{V}$ represent its velocity relative to the absolutely resting reference frame, m_i represent the inertial mass of the particle. According to special relativity, we have $m_i = m_{i0} / \sqrt{1 - V^2 / c^2}$. As shown before, when a particle moves in a gravitational field caused by a static object with spherical symmetry and static mass M , the gravitation acted on the particle is

$$\frac{d\vec{p}}{dt} = -GMm_0\left(1 + \frac{3L^2}{c^2 r^2}\right)\sqrt{1 - \frac{V^2}{c^2}}\,\frac{\vec{r}}{r^3} \tag{5.1}$$

Here $\vec{p} = m_i d\vec{r} / dt$, L is a constant. Based on the formula, we can definite gravitational moving mass

$$m_g = m_{g0}\sqrt{1 - \frac{V^2}{c^2}} \tag{5.2}$$

Here m_{g0} is gravitational static mass. Because the *Eötvös* type of experiments has shown that gravitational static mass is equivalent with inertial static mass, we have $m_{i0} = m_{g0} = m_0$. But in general situations when $\vec{V} \neq 0$, gravitational moving mass is not equivalent with inertial moving mass with $m_i \neq m_g$. Gravitational moving mass is biggest when object's speed is zero. When object's speed reaches light's speed, its gravitational moving mass becomes zero. The situation is just opposite to inertial mass. The result is interested that general relativity is based on the equivalent principle between gravitational mass and inertial mass. But it only indicates that gravitational static mass is equivalent with inertial static mass actually. After the Schwarzschild solution of the Einstein's theory of gravity is transformed into flat space-time for description, we reach the result that gravitational moving mass is not equivalent with inertial moving mass.

The classical Newtonian theory of gravitation describes the gravitation between two static objects actually. The theory has two defects. One is that it can not satisfy the Lorentz invariability. Another is that it can not describe small effects of gravitation such as the perihelion precession of the Mercury and so on. So we have to revise it from these two sides. By considering the fact that the validity of electromagnetic interaction theory and the comparability between the Newtonian formula of gravitation and the Coulomb formula of static electrics, if there exists unity between gravitational and electromagnetic interactions, gravitation would take the similar form of electromagnetic force, instead of that electromagnetic interaction should be coincide with gravitation described in curved space-time. It is well known that too many singularities appear in the Einstein's theory of gravitation described in curved space-time.

Therefore, we introduce the concepts of electric-like and magnetic-like gravitations. Suppose that there are two particles with static masses m_{10} and m_{20} moving in velocities $\vec{V}_1$ and $\vec{V}_2$ individually relative to the absolutely static reference frame. The electric-like gravitation, which is caused by the particle with static mass m_{20} and acted on the particle with static mass m_{10} and, is defined as

$$\vec{F}_e = -\frac{Gm_{g1}m_{g2}\vec{r}}{r^3}\left(1+\frac{3L_1^2}{c^2r^2}\right) = \frac{m_{g1}m_{g2}\vec{r}}{4\pi\varepsilon_g r^3}\left(1+\frac{3\left(\vec{V}_1\times\vec{e}_r\right)^2}{c^2\sqrt{1-V_1^2/c^2}}\right) \tag{5.3}$$

Here m_{g1} and m_{g2} is defined in Eq.(5.3), $\vec{r}=\vec{r}_1-\vec{r}_2$, $\vec{e}_r=\vec{r}/r$, ε_g is the so-called gravitational electric-like dielectric constant

$$\varepsilon_g = -\frac{1}{4\pi G} \tag{5.4}$$

If there are only two particles in the system, the angle momentum of unit mass $L_1=\left|\vec{V}_1\times\vec{r}\right|/\sqrt{1-V_1^2/c^2}$ is a constant. If there are more particles in the system, $\vec{L}_1$ is not a constant in general. Meanwhile, similar to electromagnetic theory, we define the magnetic-like gravitation as

$$\vec{F}_m = \frac{\mu_g m_{g1}m_{g2}}{4\pi}\frac{\vec{V}_1\times(\vec{V}_2\times\vec{r}\,)}{r^3} = \frac{\mu_g}{4\pi}\frac{\vec{J}_{g1}\times(\vec{J}_{g2}\times\vec{r}\,)}{r^3} = \vec{J}_{g1}\times\vec{B}_{g2} \tag{5.5}$$

$$\vec{J}_{gi} = m_{gi}\vec{V}_i = m_{i0}\vec{V}_i\sqrt{1-\frac{V_i^2}{c^2}} \tag{5.6}$$

Here $\vec{J}_{gi}$ is the density of mass flow, μ_g is the gravitational magnetic-like permeability and $\vec{B}_g$ is the intensity of magnetic-like gravitational field

$$\vec{B}_g = \frac{\mu_g}{4\pi}\frac{\vec{J}_g\times\vec{r}}{r^3} \tag{5.7}$$

Similar to electromagnetic theory, we can deduce

$$\frac{1}{\sqrt{\varepsilon_g\mu_g}} = c \tag{5.8}$$

By means of Eq.(5.4), we get

$$\mu_g = -\frac{4\pi G}{c^2} \tag{5.9}$$

It is useful to compare the intensities of magnetic-like and electromagnetic gravitations. We have

$$F_m \sim \frac{\mu_g}{4\pi}\frac{J_{g1}J_{g2}}{r^2} \sim \frac{G}{c^2}\frac{m_{g1}m_{g2}V_1V_2}{r^2} \sim F_e\frac{V^2}{c^2} \tag{5.10}$$

So when $V \ll c$, the magnetic-like gravitation can be neglected comparing with the electric-like gravitation. This is just the reason why the Newtonian theory of gravitation is quite effect without considering magnetic-like gravitations. In electromagnetic interaction, charged particle's speeds are great in general so that strong magnetic phenomena would be caused. But in the strong gravitational field with particle's speed $V \sim c$, magnetic-like gravitations can not be neglected.

In this way, we can establish the Maxwell's equations of gravitational fields in the similar form of electromagnetic theory. For a particle with static mass m_0 and velocity $\vec{V}$, we define the intensity of its electric-like gravitational field as

$$\vec{E}_g = \frac{m_g \vec{r}}{4\pi\varepsilon_g r^3} = \frac{m_0\sqrt{1-V^2/c^2}}{4\pi\varepsilon_g}\frac{\vec{r}}{r^3} \tag{5.11}$$

$\vec{E}_g$ is relative to particle's speed. This is different from the intensity of electric field. When material's mass is distributed continuously, the density function of gravitational moving mass should de defined as

$$\rho_g(\vec{r},t) = \rho_0(\vec{r},t)\sqrt{1-\frac{V(\vec{r},t)^2}{c^2}} \tag{5.12}$$

In which $\rho_0(\vec{r},t)$ is the density distributive function when material is at rest. In this case, Eq.(5.11) should be rewritten as

$$\vec{E}_g(\vec{x},t) = \frac{1}{4\pi\varepsilon_g}\int \frac{\rho_g(\vec{x}',t)\vec{r}}{r^3} d^3\vec{x}' \tag{5.13}$$

Here $\vec{r} = \vec{x} - \vec{x}'$. Similarly, we have

$$\nabla \cdot \vec{E}_g(\vec{x},t) = \frac{\rho_g}{\varepsilon_g} \tag{5.14}$$

For the intensity of magnetic-like gravitational field, in this case, we also have

$$\vec{B}_g(\vec{x},t) = \frac{\mu_g}{4\pi}\int \frac{\vec{J}_g(\vec{x}',t)\times\vec{r}}{r^3} d^3\vec{x}' \tag{5.15}$$

In which

$$\vec{J}_g(\vec{x}',t) = \rho_g(\vec{x}',t)\vec{V}(\vec{x}',t) = \rho_0(\vec{x}',t)\sqrt{1-\frac{V^2(\vec{x}',t)}{c^2}}\vec{V}(\vec{x}',t) \tag{5.16}$$

Also we have

$$\nabla \cdot \vec{B}_g(\vec{x},t) = 0 \tag{5.17}$$

Similar to electromagnetic theory, suppose that there exist the law of induction between electric-like and magnetic-like gravitational fields

$$\nabla \times \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t} \tag{5.18}$$

$$\nabla \times \vec{B}_g = \mu_g \vec{J}_g + \mu_g \varepsilon_g \frac{\partial \vec{E}_g}{\partial t} \tag{5.19}$$

The formulas (5.14), (5.17), (5.18) and (5.19) are the Maxwell's equation set of gravitational fields. In the formulas, the forms of $\vec{E}_g$ and $\vec{B}_g$ are determined by $\rho_g(\vec{x},t)$ and $\vec{J}_g(\vec{x},t)$. Comparing with electromagnetic theory, the only difference is that there is contraction factor of relativity in the mass density $\rho_g(\vec{x},t)$ and the mass flow density $\vec{J}_g(\vec{x},t)$. Therefore, this kind of gravitational theory is obviously invariable under the Lorentz transformation. The descriptions of gravitational and electromagnetic interactions also become consistent.

On the other hand, by means of the intensions of electric-like and magnetic-like gravitational fields, when a particle with gravitational moving mass m'_g and velocity $\vec{V}'$ moves in the gravitational field caused by another particle with gravitational moving mass m_g and velocity $\vec{V}$, the Lorentz force acted on the first particle can be represented as

$$\vec{F} = m'_g\left[\left(1+\frac{\left(\vec{V}'\times\vec{e}_r\right)^2}{c\sqrt{c^2-V'^2}}\right)\vec{E}_g + \vec{V}'\times\vec{B}_g\right] \tag{5.20}$$

Comparing with the Lorentz formula of electromagnetic theory, there exist an additional item relative to angle momentum, besides the differences of gravitational moving mass density and mass flow density.

2. The gravitational theory between objects with static masses and photons

The expression of photon's gravity is discussed below. In the spherical coordinate system, the unit vectors of directions are $\vec{e}_r$, $\vec{e}_\theta$ and $\vec{e}_\varphi$ with $\vec{e}_\varphi = \vec{e}_r\times\vec{e}_\theta = \vec{r}\times\vec{e}_\theta / r$. So Eq.(4.87) can be written as

$$\vec{F}_\varphi = \sqrt{\frac{2cV-V^2}{c^2}}\frac{GMm_0V_\varphi\left(\vec{r}\times\vec{e}_\theta\right)}{r^3V_r}\left\{1+\frac{3\left(\vec{V}\times\vec{e}_r\right)^2}{c\sqrt{2cV-V^2}}\left(1+\frac{\vec{V}_r^2}{Rc\sqrt{2cV-V^2}}\right)\right\} \tag{5.21}$$

We define longitudinal gravitational moving mass m_{gL} and transverse gravitational moving mass m_{gT} as

$$m_{gL} = m_0\sqrt{\frac{2cV-V^2}{c^2}} \qquad\qquad m_{gT} = m_0\sqrt{\frac{2cV-V^2}{c^2}}\frac{V_\varphi}{V_r} \tag{5.22}$$

By using the intensity of electric-like gravitational field, when a photon moves in a spherically symmetrical gravitational field, the electric-like gravitational force acted on the photon can be written as

$$\begin{aligned}\vec{F}_e = \vec{F}_r + \vec{F}_\varphi = m_{gL}\vec{E}_g&\left(1+\frac{\left(\vec{V}\times\vec{e}_r\right)^2}{c\sqrt{2cV-V^2}}\right)\\ &+ m_{gT}\left(\vec{E}_g\times\vec{e}_\theta\right)\left\{1+\frac{3\left(\vec{V}\times\vec{e}_r\right)^2}{c\sqrt{2cV-V^2}}\left(1+\frac{\vec{V}_r^2}{Rc\sqrt{2cV-V^2}}\right)\right\}\end{aligned} \tag{5.23}$$

In which $\vec{E}_g$ is determined by Eq.(5.11). If the center mass has a moving velocity, the corresponding magnetic-like gravitation should be added. Because the sun's velocity is very small, according to Eq.(5.10) with $F_m / F_e \sim V / c$, the magnetic-like gravitation can be neglected. So in the weak field of the sun, photon's acceleration can still be represented by Eq.(4.74). At last, for general situations, we can write the Lorentz formula of gravitation in a universally form

$$\vec{F}_g = m_{gL}\left\{\left(1+\frac{\left(\vec{V}\times\vec{e}_r\right)^2}{c\chi}\right)\vec{E}_g + \vec{V}\times\vec{B}_g\right\} + m_{gT}\left(\vec{E}_g\times\vec{e}_\theta\right)\left\{1+\frac{3\left(\vec{V}\times\vec{e}_r\right)^2}{c\chi}\left(1+\frac{\vec{V}_r^2}{Rc\chi}\right)\right\} \tag{5.24}$$

For the particles with static masses, we have $\chi = \sqrt{c^2-V^2}$, $m_{gL} = \chi m_0$ and $m_{gT} = 0$. For the photons with zero mass, we have $\chi = \sqrt{2cV-V^2}$, $m_{gL} = \chi m_0$ and $m_{gT} = m_{gL}V_\varphi / V_r$.

What we discuss above is the theory described in the absolutely resting reference frames. By the Lorentz coordinate transformation and velocity transformation in special relativity, we can describe the theory in another inertial reference frame moving relative to the absolutely resting reference frame. We discuss the transformation of gravitational moving mass below. Suppose that the sun's absolute velocity is $\vec{V}_2$ and the planet's absolute velocity is $\vec{V}_1$ relative to absolutely resting reference frame individually.

Because these two velocities are small, for simplification, we only use the Galilei's ruler of velocity transformation. So the planet's velocity relative to the sun can be written as $\vec{V} = \vec{V}_1 - \vec{V}_2$. When the sun is considered at rest, its static mass can be considered to be equivalent to $M_0' = M_0\sqrt{1 - V_2^2 / c^2} = M_g$ and the planet's static mass can also be considered to be equivalent to m_0'. So the planet's gravitational moving mass can be considered to be equivalent to

$$m_g = m_0'\sqrt{1 - \frac{V^2}{c^2}} \qquad\qquad m_0' = m_0\frac{\sqrt{1 - V_1^2 / c^2}}{\sqrt{1 - V^2 / c^2}} \tag{5.25}$$

By using relation $\vec{V}_1 = \vec{V} + \vec{V}_2$ in Eq.(5.3), we can transform Eq. (5.3)into Eq. (5.1) and get the approximate formula of electric-like gravitation in the reference frame in which the sun is considered at rest. In this case, the sun's magnetic-like gravitational can be regarded as zero approximately. Of cause, if two object's speeds are great, the addition formula of velocity in special relativity and the magnetic-like gravitation should be taken into account.

Meanwhile, this kind of gravitational theory has some natures below.

1. In this theory, quantization of gravitational field can be carried out in the similar form of electromagnetic field. Photon's spin is 1 instead of 2.

2. Similar to electromagnetic theory, this gravitational theory may be renormalizable. So it may provide a simplest foundation for the unified theory of four forces.

3. The energy momentum tensors of gravitational fields can be also defined well as that done in the electromagnetic fields. The difficulty existing in general relativity can be avoided.

4. There exist dipole radiations of gravitational waves in this theory as that in electromagnetic theory. According to general relativity, the lowest order of gravitational radiation is the fourth order. There exist no dipole radiations. This point is one of biggest differences between two theories, which can be used to decide which one is alight. At present, we only use quadrupole resonance apparatus to detect gravitational waves but find nothing. It may be more effective to use dipole resonance apparatus to do it.

It is useful to estimate the radiation strength of gravitational wave in the theory. Similar to electromagnetic theory, when a particle with static mass m moves in a speed $V << c$, the power of its gravitational radiation is

$$P_g = \frac{m^2 a^2}{6\pi\varepsilon_g c^3} \tag{5.26}$$

Here a is particle's acceleration. So for an electron, the ratio of electromagnetic radiation and gravitational radiation is the same as that in general relativity with $P_e / p_g \approx e^2\varepsilon_g / \left(m^2\varepsilon_e\right) \approx 4 \times 10^{44}$

Section 6 Application on Astrophysics and Cosmology

1. The new red-shift formula of gravitation and quasar's big red-shift

The red shift problem is discussed below. According to general relativity, gravitational field would cause time delay so that light's frequency would become small and light's wave length would become longer. However, in curved space-time, photon moves along the geodesic line without the action of force. So there is no concept of potential for photon in gravitational field. The total energy of photon is equal to its dynamic energy. If we consider that the formula $E = h\nu$ is always tenable at any point of gravitational field, because energy E is a constant, ν would also be a constant in gravitational field. But this result contradicts with the Mossbauer experiments(17). The experiments show that light's frequency would change when it moves in gravitational field. In order to keep the law of energy conservation of photon and let photon's frequency change in gravitational field, we can suppose that photon's frequency is relative to photon's dynamic energy with $T = h\nu$. Let λ and ν represent the natural wave length and frequency of light emitted by atom the r point of gravitational field, λ_0 and ν_0 represent the wave length and frequency of light observed at the point $r \to \infty$ outside gravitational field, we have $V = \lambda\nu$ and $c = \lambda_0\nu_0$ The law of energy conservation of photon can be written as

$$h\nu + U(r) = h\nu_0 = m_0c^2 \qquad \text{or} \qquad \nu = \nu_0 - U(r)/h \tag{6.1}$$

Within the region $r > \alpha$ we have $V < c$. In fact as we known that light's speed in general medium is less than its speed in vacuum. That is to say, when there is interaction, light's speed would become slow. This is a basic physical fact. On the other hand, in the practical experiments of red shift of spectrum, what measured is actually light's wave length. Owing to red shift, we have $\lambda_0 > \lambda$ and $\nu_0 < \nu$. So by considering Eqs.(4.52) and (4.93), the red shift should be defined as

$$Z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{c}{V}\frac{\nu}{\nu_0} - 1 = \frac{1}{1-\alpha/r}\left[1 - \frac{1}{4}\left(\frac{\alpha}{r}\sqrt{1-\frac{\alpha^2}{r^2}} + arcsin\frac{\alpha}{r}\right)\right] - 1$$

(6.2)

Under the condition of weak field $\alpha / r << 1$, the formula becomes

$$Z \approx \left(1 + \frac{\alpha}{r}\right)\left(1 - \frac{\alpha}{2r}\right) - 1 = \frac{GM}{c^2 r} \tag{6.3}$$

It is the same as that in general relativity. But in weak field, the results are completely different. For example, when $x \to 1$, we have $Z \to 1$ according to general relativity. Bu according to Eq.(4.94), we have $Z \to \infty$ with infinite red shift. So Eq.(4.94) can be used to calculate quasar's red shift. As we known that the physical mechanics of the big red shifts of quasars is still an enigma at present. If the red shifts are considered to be the effect of cosmology caused by the recessive speed, quasars would be at very distant places. In this case, the brightness of quasars or the mechanics of energy source becomes a problem.

Though the accretion disc theory of supermassive black holes is used to explain the energy problem of quasars at present, the existence of black holes with space-time singularity is still a unverified problem. In fact, as reported in NewScientist in July，2006, Rudolph Schild and his team in the Harvard-Smithsonian Center for Astrophysics in Cambridge, Massachusetts, U.S.A found that quasar Q0957+561 is actually a very bright and compact object, in spite of what usually thought to be generated by a giant black hole devouring its surrounding matter. A well accepted property of black holes is that they cannot sustain a magnetic field of their own. But observations of quasar Q0957+561 indicate that the object powering it does have a magnetic field(18). For this reason, rather than a black hole, this quasar would contain something called a magnetospheric eternally collapsing object (MECO).

If the centers of quasars are not black holes, there difficulty is still exist to explain their energy source. But the problem of big red shift still exists. Let quasar's radium is r and take $\alpha / r = 0.90$, we have $Z = 5.49$ from Eq.(4.94). Suppose quasar's mass is $M = 10^{40} Kg$, its radium is $r = 1.65 \times 10^{13} m$, we get its density $\rho = 0.532 Kg / m^3$. It is only 3.80×10^{-4} times of the sun's density $\rho = 1.40 \times 10^3 Kg / m^3$. This kind of density is not too big and we have no any difficulty to explain the material states of quasars.

2. Velocity and Acceleration of the Universal Expansion

At first, let's state the origin of the problems of the universal accelerating expansion and dark energy briefly. Based on the Einstein's equation of gravitational field, we can deduce the Friedman equation of cosmology containing cosmic constant or effective material density

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho - 2\rho_{eff}\right) \tag{6.3}$$

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\left(\rho + \rho_{eff}\right) \tag{6.4}$$

Here ρ is the universal material density at arbitrary moment t and ρ_{eff} is the effective density relative to vacuum and cosmological constant. Let ρ_m represent the material density at present time t_0, based on the formula, the relation between luminosity distance $d_L = r(1+Z)$ and red-shift Z in the process of the Universal expansion can be deduced[19] :

$$H_0 d_L = \frac{1+Z}{\left|\Omega_k\right|^{1/2}} \sin n \left|\Omega_k\right|^2 \int_0^z \frac{dz}{\left(1+x^2\right)\left(1+\Omega_0 x\right) - x(2+x)\Omega_{eff}} \tag{6.5}$$

Here $\Omega_k = 1 - \Omega_m - \Omega_{eff}$, $\Omega_m = \rho_m / \rho_c$, $\Omega_{eff} = \rho_{eff} / \rho_c$, ρ_c is the critical density.

The observations of the high red-shift type Ia supernovae verified the departure from the Hubble linear relation of distance---red-shift[20]. The best fit is $\Omega_{eff} = 0.70$ and $\Omega_m = 0.30$ if our universe is considered to be flat with $\Omega_m + \Omega_{eff} = 1$. So the Universal expansion seems to be accelerated at present and the concept of dark energy with repulsive action has to be introduced to explain the result. Now it becomes one of focus problems in physics and astronomy how to understand dark energy in theory and find it in experiments.

It seems to be a common idea that only general relativity could provide a proper foundation for the discussion of cosmology at present. However, it was pointed by E. A. Milne in 1943 that the Newtonian formula of gravity could also be used to describe the expansion of the universe[21]. The motion equation of the universal expansion deduced from the Newtonian theory was similar to that from general relativity, except there is no the item containing cosmic constant. It is proved below that when the formula (26) is used to describe the universal expansion, the revised Hubble formula can be deduced and the departure from the linear relation of distance---red-shift observed in the high red-shift type Ia supernovae can be explained well. Because there is no repulsive force in this theory, the hypotheses of the universal accelerating expansion and dark energy become unnecessary.

Suppose there is a medium sphere with radius R, density ρ. The static mass of sphere is $M_0 = 4\pi\rho R^3/3$. According to the Newtonian theory, the gravitation force acted on an object with static mass m_0 located at the point r outside or inside the sphere are individually[22] :

$$F = -\frac{GM_0 m_0}{r^2} \qquad r > R \tag{6.6}$$

$$F = -\frac{GM_{0r} m_0}{r^2} \qquad r < R \tag{6.7}$$

Here $M_{0r} = 4\pi\rho\, r^3 / 3$ is the static mass of sphere with radius r. The formulas indicate that when mass m_0 is located outsider the sphere with $r > R$, the gravitation acted on it is equal to that when the spherical mass is centralized at the spherical center. When mass m_0 is located inside the sphere with $r < R$, the gravitation acted on it is only relative to the spherical mass M_{0r}, having nothing to do with the mass distributed outside the radius r. It is obvious that when $R \to \infty$, the conclusion above is still tenable. We would show below that the conclusion also holds for the revised Newtonian formula (26) when object's angle velocity $\vec{L} = 0$.

In order to describe the universal expansion simply and properly, we need to establish a proper reference frame. Thought special and general relativities deny the existence of the absolutely resting reference frame, the big-bang cosmology actually implicates the existence of this kind of special reference frame. In light of the current viewpoint, the universe originated from a primordial big-bang. The big-bang means the existence of an original point. We can take this point as the original point to establish a static reference frame, called as the universal big-bang reference frame. In the expansive process of the universal, all celestial bodies and material are considered to move relative to this reference frame. In fact, in the 1960's, astronomers found the spatial anisotropy of microwave background radiation. If the reference frame in which microwave background radiation was isotropic was taken as the resting reference frame in the process of the universal expansion, observations showed that the earth was moving in a speed $300 Km \cdot s^{-1}$ towards to the directions of right ascension $1^h.5 \pm 0^h.4$ and declination $0.^0 2 \pm 7^0$ [(5)]. In 1999, the anisotropy detector of microwave background radiation (WMAP) found anisotropy in a higher precision[(6)]. In 2002, physicists found the anisotropy of radio waves eradiated by radio galaxy in the direction of the earth's motion by using array radio telescopes (VLA)[(7)]. This kind of anisotropy can also be explained by the Doppler effect of the earth's motion. So by means of these measurements of spatial anisotropy of microwave background radiation, we can already determine the orientation of the universal big-bang reference and the motions of other celestial bodies relative to it. Only by the restriction of the special and general principles of relativity, we now have no enough courage to admit it. In the following discussion, by the consideration of logical rationality, simplification and applicability, we study the problem of cosmology based on the universal big-bang reference frame.

Suppose that the universe expands along the radius direction. In the expansion process, the angle momentum $\vec{L}$ of object is equal to zero. We discuss the problems by the method of stage by stage approximation. Suppose again that an object is located at the point r in the big-bang reference frame, its velocity V_r satisfies (21) approximately at first. We consider a spherical shell with radius R of which the center is just at the original point of the big-bang reference frame. Let σ be the mass density of spherical shell. Meanwhile, there is an object located at the point $r > R$ with static mass m_0 and velocity $\vec{V}_r$ along the radius. We calculate the gravitation that the spherical shell acts on the object. Because R is a constant at a certain moment, the formula (6.4) is still effective as long as gravitational static masses are substituted by gravitational moving masses. Because the static mass of spherical shell is $M_{0\sigma} = 4\pi\sigma\, R^2$, according to (5.2), we have the gravitation that the spherical shell acts on the object

$$F_\sigma = -\frac{Gm_0 4\pi\sigma\, R^2}{r^2}\sqrt{1-\frac{V_R^2}{c^2}}\sqrt{1-\frac{V_r^2}{c^2}}$$

$$= -\frac{Gm_0 4\pi\sigma\, R^2}{r^2}\frac{\sqrt{1-V_r^2/c^2}}{\sqrt{1+\alpha_R/R}} = -\frac{GM_\sigma m}{r^2} \qquad (6.8)$$

$$M_\sigma = \frac{4\pi\sigma\ R^2}{\sqrt{1+\alpha_R/R}} = 4\pi\sigma\ R^2\sqrt{1-\frac{V_R^2}{c^2}} \qquad\qquad m = m_0\sqrt{1-\frac{V_r^2}{c^2}} \tag{6.9}$$

Here $\alpha_R = 2GM_{0R}/c^2 = 8G\pi\rho_0 R^3/(3c^2)$, $\alpha_r = 2GM_{0r}/c^2 = 8G\pi\rho_0 r^3/(3c^2)$. M_{0R} and M_{0r} are the static masses of spheres with the same densities ρ_0 but different radius R and r. Because R is a constant for a sphere shell, the formula (6.8) represents the gravitation acted on the moving object and caused by the spherical shell when its gravitational moving mass is considered to centralize at the spherical center. Let $b = 8G\pi\rho_0/(3c^2)$, $x = \alpha_R/R = bR^2$. By substituting both into (6.8) and taking the integral over R, we get the total gravitation that the expensive sphere with radius $R = r$ acts on an object with static mass m_0 and velocity V_r located on the spherical surface

$$F = -\frac{Gm_0 4\pi\sigma}{r^2}\sqrt{1-\frac{V_r^2}{c^2}}\int_0^r \frac{R^2 dR}{\sqrt{1+bR^2}}$$

$$= -\frac{GM_{0r}m_0}{r^2}\frac{3}{2}\frac{\sqrt{x+x^2} - ln\left(\sqrt{x}+\sqrt{1+x}\right)}{x^{3/2}}\sqrt{1-\frac{V_r^2}{c^2}} \tag{6.10}$$

On the other hand, we have

$$F = \frac{d}{dt}\frac{m_0 V_r}{\sqrt{1-V_r^2/c^2}} = \frac{m_0\ddot{r}}{\left(1-V_r^2/c^2\right)^{3/2}} \tag{6.11}$$

From both formulas above, we obtain the acceleration of an object on the spherical surface in the processes of the universal expansion

$$\ddot{r} = -\frac{GM_r}{r^2}\left(1-\frac{V_r^2}{c^2}\right)^2 = -\frac{GM_{0r}}{r^2}Q(x) \tag{6.12}$$

$$M_r = M_{0r}\frac{3}{2}\frac{\sqrt{x+x^2} - ln\left(\sqrt{x}+\sqrt{1+x}\right)}{x^{3/2}} \qquad Q(x) = \frac{3}{2}\frac{\sqrt{x+x^2} - ln\left(\sqrt{x}+\sqrt{1+x}\right)}{x^{3/2}(1+x)^2} \tag{6.13}$$

Here M_r is the gravitational moving mass of expansive sphere and M_0 is the static mass of sphere.

By the same consideration, it is easy to understand that when $R > r$ and $\vec{L} = 0$, the resultant gravitation caused by the spherical shell, acted on an object which is located inside the spherical shell, is also zero. But it is unnecessary for us to discuss any more here.

So when we discuss the problems of cosmology based on the big-bang reference frame, the gravitation acted on an object which is located at point r is only relative to the mass contained in the spherical shell with radius r, having nothing to do with the total mass of the universe, no mater whether the universe is finite or infinite. Suppose that the total mass contained in the spherical shell is M_0, it is enough for us only to consider the gravitation caused by M_0, acted on the object located on the surface of sphere. Because mass M_0 is finite, when the spherical radius $r \to \infty$ in the process of the universal expansion, we have $x \to 0$. When the spherical radius $r \to 0$ in the process of the universal contraction, we have $x \to \infty$. It is easy to prove the following limitations

$$\lim_{x\to 0} Q(x) = \lim_{x\to 0}\frac{3}{\sqrt{1+x}(1+x)^2(3+2x)} = 1 \tag{6.14}$$

$$\lim_{x\to\infty} Q(x) = \lim_{x\to 0} \frac{3}{\sqrt{1+x}(1+x)^2(3+2x)} = 0 \tag{6.15}$$

By the numerical calculation, it can be known that we always have $Q(x) > 0$ and $\ddot{r} \le 0$ within the region $0 < x < 1$. So the expansive speed of the universe is always decreased, that is to say, no the universal accelerating expansion actually. In order to know object's velocity in the expansive process, by considering relation $\ddot{r} = dV_r / dt = V_r dV_r / dr$ and taking the integral of (36), we have

$$V_r^2 - V_0^2 = 3GM_0 \int_{x_0}^{x} \frac{\sqrt{x' + x'^2} - ln\left(\sqrt{x'} + \sqrt{1+x'}\right)}{x'^{3/2}(1+x')^2} dx' \tag{6.16}$$

It is difficult to complete the integral. But we can do approximate calculation. We have following developing formulas within the region $-1 < x < 1$

$$\frac{1}{\sqrt{1+x}} = 1 - 0.50x + 0.38x^2 - 0.31x^3 + 0.27x^4 - 0.25x^5 \cdots \tag{6.17}$$

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 \cdots \tag{6.18}$$

By substituting (6.17) into (6.10) and considering (6.11) and (6.18), we get at last

$$\ddot{r} = -\frac{GM_0}{r^2}\left(1 - \frac{2.30\alpha}{r} + \frac{3.76\alpha^2}{r^2} - \frac{5.32\alpha^3}{r^3} + \frac{6.95\alpha^4}{r^4} - \frac{8.64\alpha^5}{r^5}\cdots\right) \tag{6.19}$$

$$V_r^2 = \frac{2GM_0}{r}\left(1 - \frac{1.15\alpha}{r} + \frac{1.25\alpha^2}{r^2} - \frac{1.33\alpha^3}{r^3} + \frac{1.39\alpha^4}{r^4} - \frac{1.44\alpha^5}{r^5}\cdots\right) + A \tag{6.20}$$

Let $V_r = 0$ when $r \to \infty$, we have integral constant $A = 0$. Then considering (6.20) as the more accurate speed of an object located on the expansive spherical surface, substituting it into (6.10) and doing the second time of calculation in light of the same procedure, we get the second approximate results of the acceleration and velocity. Let $V_r \to V$, we have at last

$$\ddot{r} = -\frac{GM_0}{r^2}\left(1 - \frac{2.30\alpha}{r} + \frac{4.09\alpha^2}{r^2} - \frac{6.62\alpha^3}{r^3} + \frac{8.74\alpha^4}{r^4} - \frac{11.55\alpha^5}{r^5}\cdots\right) \tag{6.21}$$

$$V^2 = \frac{2GM_0}{r}\left(1 - \frac{1.15\alpha}{r} + \frac{1.36\alpha^2}{r^2} - \frac{1.57\alpha^3}{r^3} + \frac{1.75\alpha^4}{r^4} - \frac{1.93\alpha^5}{r^5}\cdots\right) \tag{6.22}$$

It can be seen that the differences only appear on the third and later items comparing with (6.19) and (6.20).

3. The revised Hubble formula and the inexistence of dark energy

We now discuss the problem of the universal expansion based on (6.21). From it we can obtain

$$V = H_0 r \sqrt{1 - 1.15\left(\frac{H_0 r}{c}\right)^2 + 1.36\left(\frac{H_0 r}{c}\right)^4 - 1.57\left(\frac{H_0 r}{c}\right)^6 + 1.75\left(\frac{H_0 r}{c}\right)^8 - 1.93\left(\frac{H_0 r}{c}\right)^{10} \cdots} \tag{6.23}$$

Here $H_0 = \sqrt{8\pi G\rho/3}$ is the Hubble constant. When $\alpha / r = (H_0 r / c)^2 << 1$, we get the Hubble formula from the formula above

$$Z = \frac{V}{c} = \frac{H_0}{c} r \tag{6.24}$$

It is noted that according to the current definition, the Hubble constant $H_0 = \sqrt{8\pi G \rho_c / 3}$. Here ρ_c is the critical material density, in spite of the material density ρ. By the astronomy observation, we can take $H_0 = 65Km/(s \cdot Mpc) = 2.1 \times 10^{-18} / s$ at present and get $H_0 / c = 7 \times 10^{-27} / m$. So (6.24) is only suitable for the situation with $r < 10^{25} m$.

Thought the Hubble formula (6.24) is defined in the big-bang reference frame, it is easy to prove that the formula is also suitable for the observation on the earth reference frame. As shown in Fig.6.1, suppose that r_1 and $\vec{V}_1$ are the distance and velocity of the earth relative to the big-bang reference frame, r_2 and $\vec{V}_2$ are the distance and velocity of a certain celestial body relative to the big-bang reference frame located at arbitrary direction θ, r and $\vec{V}$ are the distance and velocity of the celestial body relative to the earth, we have $V_1 = H_0 r_1$, $V_2 = H_0 r_2$ and obtain

$$V = \sqrt{V_1^2 + V_2^2 - 2V_1 V_2 \cos\theta} = H_0 \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta} = H_0 r \tag{6.25}$$

So the Hubble distance--redshift linear relation between the earth and celestial bodies at arbitrary direction still holds when the revised items are neglected. When the observers on the earth measures the distant celestial bodies with red-shift $Z >> 10^{-3}$, the distance between the earth and the original point of the big-bang reference frame can be neglected, so that the red-shift observed on the earth can be considered as that observed at the big-bang reference frame. We discuss the problems of the high red-shift type Ia supernova in this way below.

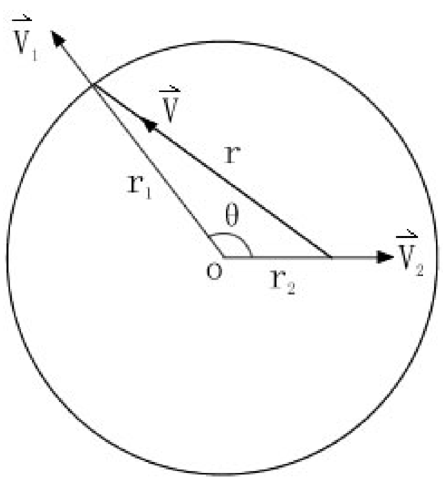


Fig. 6.1 The Hubble redshift --distance relation between the earth and the celestial bodies at arbitrary space directions

Fig. 6.2 is the Hubble diagram of the type Ia high red-shift supernovae (Ricss A. G., et al 1998) [20]. In the figure, $m - M = 5 \log d_L + 25$, luminosity distance $d_L = r(1+Z)$ is the unit of Mpc. The relation between distance and red-shift departures the Hubble linear relation when $Z > 0.1$. The first full curve is the result of the Einstein—de Sitter modal with $\Omega_m = 1,\ \Omega_\lambda = 0$. Because the modal does not fit the observations, the Einstein—de Sitter modal is clearly ruled out. The second dotted curve is the result of the so-called empty universe with $\Omega_m = 0,\ \Omega_\lambda = 0$. It is also not a good fit. The observational results of distant supernovae lie below it. From the figure（as well as that from Perlmutter S., et al, 1998）, it is concluded that the universe is being accelerated at present day. In order to explain the result, the hypothesis of dark energy has to be introduced.

On the other hand, we write the revised Hubble formula (48) as $V = Hr$ with

$$H = H_0\sqrt{1 - 1.15\left(\frac{H_0 r}{c}\right)^2 + 1.36\left(\frac{H_0 r}{c}\right)^4 - 1.57\left(\frac{H_0 r}{c}\right)^6 + 1.75\left(\frac{H_0 r}{c}\right)^8 - 1.93\left(\frac{H_0 r}{c}\right)^{10} \cdots} \quad (6.26)$$

Here H is not a constant again when α / r can not be neglected. The general relation between speed and red-shift is

$$Z = \sqrt{\frac{1 + V/c}{1 - V/c}} - 1 \quad (6.27)$$

Substitute (6.23) into the formula above, the relation between real distance and red-shift can be obtained. Then by the definition $d_L = r(1+Z)$, the relation between luminosity distance and red-shift is also obtained as shown in Fig.6.2. We take $r = 10^{25}m \sim 10^{26}m$ in the figure. The result shows that if when $H_0 = 65 Km \cdot s^{-1} \cdot Mpc^{-1}$, the curve is almost similar to that of the empty universe in the current theory. When $H_0 = 60 Km \cdot s^{-1} \cdot Mpc^{-1}$, we get the third dotted curve which is a good fit of the observational results of distant supernovae. The forth dotted curve for $H_0 = 55 Km \cdot s^{-1} \cdot Mpc^{-1}$ is also not a good fit comparing with the third curve. The result shows that we may take $H_0 = 60 Km \cdot s^{-1} \cdot Mpc^{-1}$. In this way, the expansive universe is only controlled by gravitation, no repulsion force to exist. So there exists no accelerating expansion of the universe and the hypothesis of dark energy becomes unnecessary.

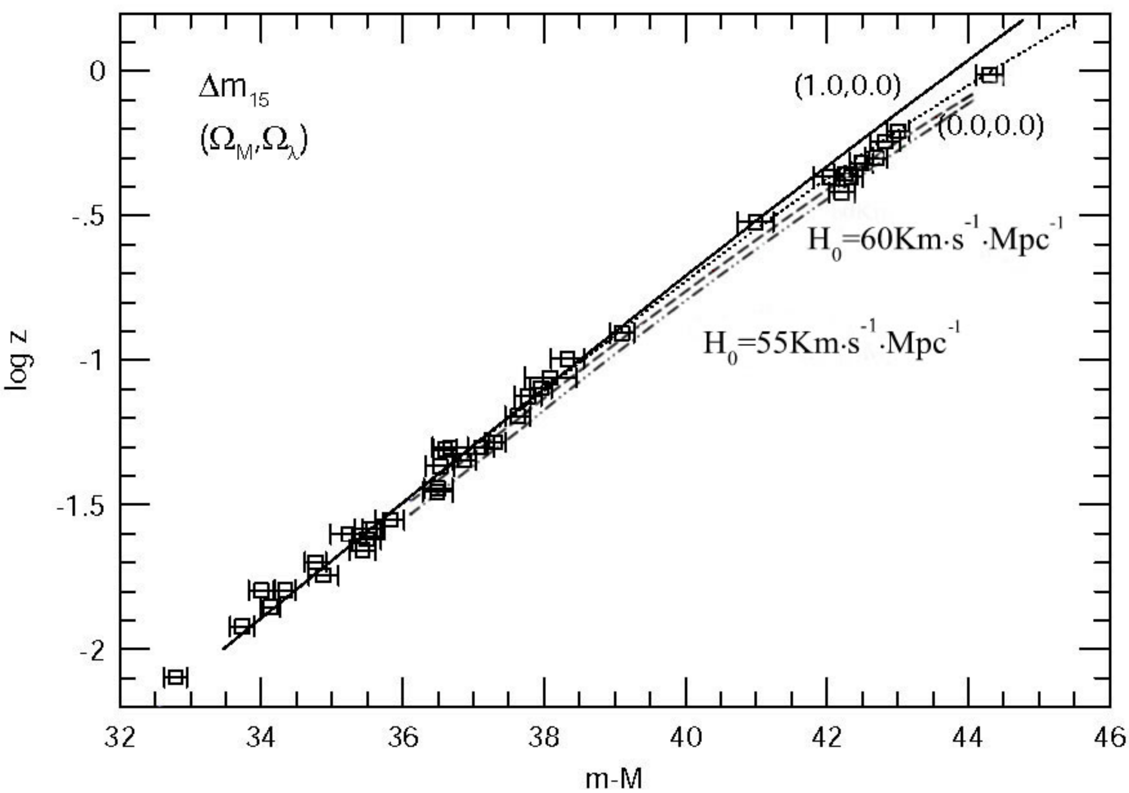


Fig.2 Comparison with the Hubble diagram of the Ia type high red-shift supernovae search（Ricss A.G., et al 1998）

From $H_0 = 60Km \cdot s^{-1} \cdot Mpc^{-1}$, we can get $\rho_0 = 6.74 \times 10^{-27} Kg \cdot m^{-3}$. Let the density of baryon material be ρ_1 and the density of non-baryon material be ρ_2, we have $\rho_0 = \rho_1 + \rho_2$. By the astronomic observation and theoretical estimation, it seems proper to take $\rho_2 / \rho_1 \sim 6$. We have $\rho_1 \approx 10^{-27} Kg \cdot m^{-3}$ and $\rho_2 \approx 5.74 \times 10^{-27} Kg \cdot m^{-3}$ for the current universe, that is to say, we still need the hypothesis of dark material in this paper.

4. The problem of the universal age

Suppose that an object located at point r_0 at time t_0 moves to another point r at time t_0 in the process of the universal expansion. If the revised items of the Newtonian gravitation are not considered, we have $V = dr / dt \sim \sqrt{2GM_0 / r}$. By taking integral, we get

$$t - t_0 = \int_{r_0}^{r} \sqrt{\frac{r'}{2GM_0}} dr' = \frac{2}{3\sqrt{2GM_0}} \left(r^{3/2} - r_0^{3/2} \right) \tag{6.28}$$

Let $r_0 \to 0$ when $t_0 = 0$, and substitute $M_0 = 4\pi\rho\, r^3 / 3$ into the formula above. By taking $H_0 = 60Km \cdot s^{-1} \cdot Mpc^{-1}$, we get

$$t = \frac{1}{\sqrt{6G\pi\rho}} = \frac{2}{3H_0} = 1.09 \times 10^{10} y \tag{6.29}$$

This is just the result in the current theory for the flat universe with curvature constant $k = 0$. This age of the universe is less than that of the spherical star cluster. If the revised items are considered, let $x = \alpha / r$, we have

$$t = \frac{\alpha}{c} \int_{x}^{\infty} \frac{dx'}{x'^{5/2} \sqrt{1 - 1.15x' + 1.36x'^2 - 1.57x'^3 + 1.75x'^4 - 1.93x'^5}} \tag{6.30}$$

The integral is difficult. Because the item in the radical sign is less than 1, we have $t > 2/(3H_0)$. Take $x = 0.7$ （$r = 1.29 \times 10^{26} m$）, $\sqrt{1 - 1.15x + 1.36x^2 - 1.57x^3 + 1.75x^4 - 1.93x^5} = 0.65$, by the approximate estimate, we can let

$$t \approx \frac{\alpha}{0.65c} \int_{x}^{\infty} \frac{dx'}{x'^{5/2}} = \frac{2}{3 \times 0.65 H_0} \approx 1.68 \times 10^{10} y \tag{6.31}$$

In this case, the universal age is similar to that of the spherical star cluster. Because the radius of the universe can be considered to be large than $1.29 \times 10^{26} m$, we have $t > 1.68 \times 10^{10} y$, so there exists no problem about the age of the universe according to this theory. Besides, we have decelerating parameter

$$q = -\frac{\ddot{r} r}{\dot{r}^2} = \frac{1}{2} \frac{1 - 2.30\alpha / r + 4.09\alpha^2 / r^2 - 6.62\alpha^3 / r^3 + 8.74\alpha^4 / r^4 - 11.55\alpha^5 / r^5}{1 - 1.15\alpha / r + 1.36\alpha^2 / r^2 - 1.57\alpha^3 / r^3 + 1.75\alpha^4 / r^4 - 1.93\alpha^5 / r^5} \tag{6.32}$$

When $\alpha / r \to 0$ we have $q = 1/2$. This is just the situation of critical modal with $k = 0$ in the current theory of cosmology. From the observation of the spatial anisotropy of microwave background radiation, we know that the earth's speed is about $\dot{r} \approx 3 \times 10^5 m \cdot s^{-1}$ relative to the big-bang reference frame. So the distance between the earth and the original point of the big-bang reference frame is about $r \approx 1.67 \times 10^{23} m$ according to (6.25). We can calculate the earth's acceleration $\ddot{r} \approx -\dot{r}^2 / (2r)$ $= -2.69 \times 10^{-13} m \cdot s^{-2}$ relative to the big-bang reference frame.

5. The motion equation of the universal expansion

Lel's discuss how to transform the formula (6.21) and (6.22) into the form of the current cosmology. Let $\bar{r}(t) = R(t)r$, r is the coordinate of medium, $R(t)$ is the scalar factor of the expanding universe. Let t_0 represent the current time, we have $\bar{r}(t_0) = r$, $R(t_0) = 1$. In this way, the formula (6.21) can be written as

$$\ddot{R} = -\frac{4\pi G}{3}\rho R\left(1 - \frac{6.13\pi G}{c^2}\rho R^2 r^2 + \frac{29.01\pi^2 G^2}{c^4}\rho^2 R^4 r^4 - \frac{125.53\pi^3 G^3}{c^6}\rho^3 R^6 r^6 \cdots\right) \quad (6.33)$$

Because of $\rho(t)R^3(t) = \rho(t_0)R^3(t_0) =$ constant, we can let

$$\frac{6.13\pi G}{c^2 R(t_0)}\rho(t)R^3(t)r^2 = \frac{6.13\pi G}{c^2}\rho(t_0)R^2(t_0)r^2 = b_1' \quad (6.34)$$

$$\frac{29.01\pi^2 G^2}{c^4 R^2(t_0)}\rho^2(t)R^6(t)r^4 = \frac{6.13\pi G}{c^2}\rho^2(t_0)R^4(t_0)r^4 = b_2' \quad (6.35)$$

$$\frac{125.53\pi^3 G^3}{c^4 R^3(t_0)}\rho^3(t)R^9(t)r^6 = \frac{6.13\pi G}{c^2}\rho^3(t_0)R^6(t_0)r^6 = b_3' \quad (6.36)$$

Here b_i' are dimensionless parameters represented by the current time. Let $R(t_0) = R_0$, the formula (6.33) can be written as again

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\rho\left(1 - \frac{b_1' R_0}{R} + \frac{b_2' R_0^2}{R^2} - \frac{b_3' R_0^3}{R^3} + \cdots\right) \quad (6.37)$$

By taking the integral, we obtain

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{b_1}{R} + \frac{b_2}{R^2} - \frac{b_3}{R^3}\cdots\right) \quad (6.38)$$

Here $b_1 = b_1' R_0 / 2$, $b_2 = b_2' R_0^2 / 3$, $b_3 = b_3' R_0^3 / 4 \cdots$. Comparing with the standard form in the current theory (6.4), the item relative to the universal constant is replaced by the items containing b_i. In the current cosmology, k is a curvature constant. But in (6.38), k is an integral constant and we can take $k = 0$ actually according to (6.22). The result corresponds to the flat universe in the current cosmology. Let $\rho_1' = -\rho(-b_1' R_0 / R + b_2 R_0^2 / R^2 - \cdots)/2$, $\rho_2' = \rho(-b_1 / R + b_2 / R^2 - \cdots)$, we can write (6.37) and (6.38) as

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho - 2\rho_1') \qquad \left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}(\rho + \rho_2') \quad (6.39)$$

It is obvious that the difference between two theories is only on the different definition of ρ_{eff} .In the current cosmology, we have $\rho_{eff} = \rho_\lambda + \rho_{vac}$, in which ρ_{vac} is the material density of vacuum and ρ_λ is the material density corresponding to the universal constant. Meanwhile, ρ_{eff} is considered to be a constant having noting to do with space-time coordinates in the current theory. In order to coincide with the observation results of Ia supernovae, we have to suppose $\rho - 2\rho_{eff} < 0$. Therefore, we have to think that our universal is being acted by the repulsive force and is doing the accelerating expansion at present. According to this paper, however, ρ'_k are not constants and we always have $\rho - 2\rho'_k > 0$.There exists no the accelerating expansion of the universe. We do not need the hypothesis of dark energy. In fact, there exists a sharp contradiction in the problem of cosmic constant which has puzzled physical circle for a long time. By means of the method in this paper, we can rid of this problem thoroughly.

3. The possibility to explain the Pioneer Anomaly

NASA had lunched a series of spacecrafts such as the Pioneers 10 and 11 for the exploration of `the Jupiter and Saturn since 1970's last century. Recently, it is founded that most of the spacecrafts departure their orbits. The orbits are calculated by the Einstein's theory of gravitation. An additional constant acceleration is found for them with $a_p = (8.74 \pm 1.33) \times 10^{-10} m/s^2$ according to the data up to date(23). The direction of a_p is towards the sun. After all of possible factors, which would affect the orbits of the spacecrafts, were excluded, scientists in NASA affirmed that there exists the unexplained Pioneer Anomaly.

Since the Pioneer Anomaly was founded in 1998, many theories had been proposed, but none of them was satisfied(24). The orbits of the spacecrafts and the acceleration a_p are calculated actually by the PPN approximate method based on the Einstein's theory. So the Pioneer Anomaly means that the Einstein's theory of gravitation would be revised. New we discuss the possibility to explain the Pioneer Anomaly in light of this paper's theory. The accelerations of spacecrafts are calculated by using the formula below(25)

$$\vec{a}_i = \sum_{j \neq i} \frac{\mu_i (\vec{r}_j - \vec{r}_i)}{r_{ij}^3} \left\{ 1 - \sum_{k \neq i} \frac{4\mu_k}{c^2 r_{ik}} - \sum_{k \neq j} \frac{\mu_k}{c^2 r_{jk}} - \frac{3\left[(\vec{r}_j - \vec{r}_i) \cdot \vec{V}_j\right]^2}{2c^2 r_{ij}^2} + \frac{(\vec{r}_j - \vec{r}_i) \cdot \vec{a}_j}{2c^2} \right.$$
$$\left. - \frac{4\vec{V}_i \cdot \vec{V}_j}{c^2} + \frac{\vec{V}_i^2}{c^2} + \frac{2\vec{V}_j^2}{c^2} \right\} + \sum_{j \neq i} \frac{\mu_j (\vec{V}_i - \vec{V}_j)}{c^2 r_{ij}^3} \left[(\vec{r}_j - \vec{r}_i) \cdot (4\vec{V}_i - 3\vec{V}_j) \right] + \sum_{j \neq i} \frac{7\mu_j \vec{a}_j}{2c^2 r_{ij}} \tag{6.40}$$

Here $r_{ij} = |\vec{r}_j - \vec{r}_i|$, $\mu_i = GM_i$. M_i, $\vec{a}_i$ and $\vec{V}_i$ are the i object's static mass acceleration and velocity individually. If only a spacecraft moves in the static gravitational field of the sun, we can let $\vec{V}_2 = \vec{a}_2 = 0$, $\vec{r} = \vec{r}_{12}$, $\vec{V} = \vec{V}_1$ and get spacecraft 's acceleration from the formula above

$$\vec{a} = -\frac{GM\vec{r}}{r^3} \left(1 - \frac{4GM}{c^2 r} - \frac{4Gm}{c^2 r} + \frac{\vec{V}^2}{c^2} \right) + \frac{4GM (\vec{r} \cdot \vec{V}) \vec{V}}{c^2 r^3} \tag{6.41}$$

Because the PPN approximate method is also based on curved space-time actually, the acceleration shown in Eq.(6.41) can not be compared with the experiments carried on the earth before it is transformed into the result in flat space-time. So it is meaningless actually. According to the paper, we should calculate gravitational interaction among the sun, planets and spacecrafts based on Eq.(5.20). Here we only use Eq.(5.2) to show the acceleration of spacecraft. It should be

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -\frac{GM\vec{r}}{r^3} \left(1 + \frac{3L^2}{c^2 r^2} \right) \left(1 - \frac{V^2}{c^2} \right) - \frac{\dot{V}V\vec{V}}{c^2 (1 - V^2/c^2)} \tag{6.42}$$

In which V and $\dot{V} = dV/dt$ are determined by Eq.(4.22). It can be seen that the formula (6.42) is different from Eq.(6.41). The formula (6.42) is an accurate result relative to angle momentum. But Eq.(6.41) is an approximate result having nothing to do with angle momentum. For simplification, we only discuss the first item of Eq.(6.42). When $\alpha << r$, $V << c$, by remaining the items up to the order r^{-4} , we have

$$a = -\left(1 + \frac{3L^2}{c^2 r^2} - \frac{GM}{c^2 r}\right)\frac{GM}{r^2} \qquad \Delta a = -\left(\frac{3L^2}{c^2 r^2} - \frac{GM}{c^2 r}\right)\frac{GM}{r^2} \tag{6.43}$$

Here Δa is just the additional acceleration comparing with the Newtonian theory. Suppose that the spacecraft moves along the tangent direction near the solar surface in the third universal velocity $V = 4.20 \times 10^4 M/s$, so that it can escape the sun's gravitation. The sun's mass is $M = 1.99 \times 10^{30} Kg$, the angle momentum of spacecraft is $L = VR$. The additional acceleration is a positive value with $\Delta a = 5.64 \times 10^{-4} m/s^2$. The result means that the revised force is a repulsive one, instead of gravitation. When the spacecraft is on the mercurial orbit, we get $\Delta a = -1.38 \times 10^{-9} m/s^2$. If the spacecraft is on the earth's orbit, we have $\Delta a = -2.93 \times 10^{-10} m/s^2$ with the same magnitude of the Pioneer anomaly a_p . When spacecraft is on the Jupiter's orbit, we have $\Delta a = -6.38 \times 10^{-14} m/s^2$. It becomes very small. When spacecraft moves on the earth's surface in the second universal velocity $V = 1.12 \times 10^4 m/s$, we have $\Delta a = -3.41 \times 10^{-8} m/s^2$. So the earth, Jupiter and other planets would also cause additional accelerations for spacecraft. Especially, the Jupiter and Saturn's masses are quite big, the additional accelerations should be taken into account.

In the practical calculations, the magnetic-like gravitation caused by the motions of planets should also be considered. The time delay effects of radar waves emitted by spacecraft should also be calculated by means of Eq.(4.74). At last, the really strict calculations should be carried out in the absolutely reference frame. Because the multi-body problems are involved, the practical orbits of spacecrafts should be calculated by computer. By comparing the results of numerical value calculations with the practical orbits of spacecraft, we can judge whether or not the theory of this papers is more rational than the general relativity. If it is alight, we would reach a really rational theory of gravitation, and obtain a credible foundation for the unification of four interaction forces.

To sum, in order to explain the light's propagations in vacuum a hundred years ago, the hypothesis of the ether with very strange natures was putted forward. In order to eliminate the ether, Einstein established special relativity. After that, general relativity was advanced. The theories caused the idea revolution of space-time and gravitation and promoted the development of physics. A hundred year later, in order to explain so many contradictions and anomalies, we foist too many things such as cosmic constant, vacuum energy, dark energy and space-time singularity and so on into vacuum again. The situation actually indicates that we need another idea renewal on space-time and gravitation. The result would be that these concepts would be given up at last, just as that the concept of the ether was given up a hundred years ago.

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